A hidden symmetry in the sequences

”First Digits of $2^n$, $3^n$, $4^n$, $5^n$, ...”

Abstract: The sequence of the last (rightmost) digits of the integers $2^n$ is periodic:

$$(2, 4, 8, 6), (2, 4, 8, 6), ... .$$

On the contrary, the sequence of the first (leading) digits of the integers $2^n$ is chaotic:

$$2, 4, 8, 1, 3, 6, 1, 5, ... .$$

QUESTION: Does this chaotic sequence of the first digits contain the digit 7 or the digit 9? If yes, then which digit, 7, 8, or 9, appears more frequently in this sequence?

The same question can be posed for the sequences “First digits of $3^n$, of $4^n$, of $5^n$, ...”. Or, generally, we can consider sequences of more than one leading digit and ask similar questions:

- Can $2^n$ and $3^n$ both have one or more leading digits of the number $\pi = 3.1415\ldots$, or both have one or more leading digits of the number $e = 2.71828\ldots$?

- Can $2^n$ and $5^n$ both start with the same one or more leading digits and if yes, then what could these digits be? The same question for the triplet $2^n$, $3^n$, $5^n$.

It turns out that to answer to that kind of pure arithmetic questions, one needs to take into consideration special dynamical systems on a circle $S^1$ and on a torus $T^2$.

In my talk, I will associate the above sequences with dynamical systems on the circle and on the torus, and will also tell about my recent discovery of a hidden symmetry in the set of the exponents $\{ n \}$ for the sequences $2^n$, $3^n$, $4^n$, $5^n$, ... .

All the core ideas will be explained on a very elementary level, so everyone, especially students, are very welcome to attend the talk.

SNACKS IN FACULTY LOUNGE AT 3:30 PM.
EVERYONE WELCOME (EVEN IF YOU ARE UNABLE TO ATTEND THE TALK)