Impact of Implementing Error Analysis on Mathematics Proficiency in Fourth Grade

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Abstract

A large body of educational literature and research suggests that there is a deficit in students' mathematics proficiency and performance. Students often make errors when completing mathematics problems but are not capable of identifying the errors or reasons that the errors occurred. This lack of understanding can affect test scores and students' comprehension of subsequent mathematics topics and skills. This study responds to the need for improved mathematics instruction in elementary classrooms by proposing a strategy for increasing students' usage of error analysis in order to improve their mathematics accuracy and comprehension for the skills of addition, subtraction, and multiplication. The study was conducted for six weeks and included 17 fourth grade participants. The researcher utilized pre- and post-assessments and exit slips to collect data in the study. Based upon the data collected, the strategy of error analysis was effective in increasing participants' proficiency in addition, subtraction, and multiplication in fourth grade. Results revealed the mean pre-assessment score was 4.29 (*SD* =2.52) and the mean post-assessment score was 14.76 (*SD*= 1.55). Participants made an overall mean score gain of 10.47 (*SD*=2.12) from the beginning to the end of the study. This reveals that error analysis had a positive effect on participants' mathematics scores.

Keywords: elementary school, mathematics teaching, error analysis

Impact of Implementing Error Analysis on Mathematics Proficiency in Fourth Grade

There is a dilemma in the United States (U.S.) that has affected young students negatively, and it is imperative that the problem is addressed. Mathematics proficiency in elementary students has greatly decreased in the past few years, widely due to the impacts of the COVID-19 pandemic. Specifically, students on average today are approximately five months behind in mathematics due to the significant effect that the COVID-19 pandemic had on their learning (Dorn et al., 2021). It is essential to address this issue because of the importance of mathematics in elementary schools as well as in students' later lives. "Mathematics is one of the most important subjects learned in school, since mathematic achievement is one of the strongest predictors of later academic success" (Lucangeli et al., 2019, p. 578). When students come to school with a deficit in mathematics proficiency, it is vital that teachers begin instruction and intervention immediately.

Addressing needs of unique students and having all students achieving proficiency in mathematics is challenging in today's classrooms. Lang-Raad and Marzano (2019) note:

> "Unfortunately, many mathematics teachers focus on fluency and practice in ways that give students the impression that practice, drills, tips, and tricks are the focus of solving mathematics problems. Ideally, mathematics practice and fluency should illuminate the flexible nature of numbers and the different mathematical contexts that exist. (p. 45)

Students who are in fourth grade are introduced to many new, challenging concepts and skills in the mathematics content area, and if students acquire understanding of the fourth-grade standards, this will aid in their future mathematics success and level of proficiency. "Students' ability to meet fourth grade benchmarks is a strong predictor of later academic success. It is a fundamental grade to develop and master skills that prepare students for moving on to more

advanced math" (Sattem et al., 2022, p. 9). COVID-19 increased learning gaps in American schools, and, according to testing data, the greatest needs for today's students have been seen in mathematics (Sattem et al., 2022). Teachers can choose to address this deficit using various strategies and methods. One of the most worthwhile options for instructional strategies that will adequately address the deficit in mathematics proficiency in elementary students is to implement error analysis with students.

The purpose of the study was to determine the effect implementing error analysis activities will have on student understanding of fourth grade mathematics concepts. The increased learning gaps in mathematics proficiency in today's elementary school students is alarming enough to warrant further research (Sattem et al., 2022). Due to the push for educators to make an attempt to close the mathematics achievement gaps in today's elementary school students, the researcher was determined to utilize a proven effective strategy to improve mathematics instruction and student understanding. The researcher used the instructional strategy of error analysis in the study because the research on this particular strategy indicates that it is highly effective in accurately solving mathematics problems and comprehending mathematics content (Lang-Raad & Marzano, 2019. The study will offer educators research-based data about the value or non-value of implementing error analysis in mathematics instruction in a fourth-grade classroom.

Two research questions guided this study:

1. Does the use of error analysis affect participants' comprehension of mathematical procedures and skills?
2. Does the use of error analysis affect participants' accuracy when completing mathematics problems?

The study hypothesized that engaging in error analysis will improve participants' ability to comprehend mathematical procedures and skills. The study also hypothesized that correcting errors and explaining the reason for errors in mathematics will help participants to describe mathematical procedures in their own words, define key vocabulary terms, and solve future problems accurately.

In the following literature review, the author will synthesize studies and define what error analysis is and to examine the benefits of error analysis in mathematics instruction, including brain growth and positive impacts on student accuracy and comprehension. An overview of the strategy, theoretical foundations, and historical context are provided with this review. Additionally, a review of the research in favor of implementing error analysis in mathematics instruction will be analyzed, particularly the abundant research conducted by Marzano. The researcher will discuss how teachers can plan for the implementation of error analysis in elementary mathematics instruction. Lastly, the researcher will address how implementing error analysis can assist reluctant learners in feeling more confident and comfortable in their mathematics learning.

Error Analysis

Overview

Error analysis is an evidence-based instructional strategy and cognitive skill that requires students to assess errors in computation and mathematical processes, describe why the solution is incorrect, and correctly complete the problem. Marzano (2017) defines error analysis as "identifying common logical errors or common misconceptions in content; identifying common errors in the execution of a process" (Marzano et al., 2019, p. 34). Error analysis can be an addition to mathematics instruction at any grade level in order to improve student learning.

Studies have shown that students' understanding of proficiency in mathematical concepts increases when they use the strategy of error analysis (Rushton, 2018). Therefore, error analysis is capable of being an asset to mathematics instruction. "Incorrect knowledge can induce cognitive conflicts which prompt the learner to build up a coherent knowledge structure" (Rushton, 2018, p. 2). In other words, when students correct their incorrect reasoning, they are likely to comprehend material better. Several past researchers have found that "students ranging from elementary mathematics to university undergraduate medical school who, when given correctly worked examples and erroneous examples, learned more than students who only examined correctly worked examples" (Rushton, 2018, p. 3). Research supports the use of error analysis in mathematics instruction due to its success in assisting students in knowledge and skill retention.

Standards for Mathematics

The majority of school districts in the U.S. plan instruction in mathematics based upon the Common Core State Standards, which were developed to address the need for mathematics instruction in the United States to become more focused and clearer in order to improve mathematics achievement (Common Core State Standards Initiative, 2021). The Common Core State Standards guide teachers' instruction, and error analysis can be implemented as an instructional strategy in order to support students in learning the content stipulated in the standards. "Researchers found the process of explaining and justifying solutions for both correct and erroneous examples to be more beneficial for achieving learning outcomes than explaining and justifying solutions to correctly worked examples only" (Rushton, 2018, p. 2). Requiring students to analyze errors in mathematical solution paths allows them to construct practical arguments and describe the reasoning of erroneous solutions (Rushton, 2018). This addresses the

Common Core State Standards' stipulation that students must solve real-world problems and be able to not only use a procedural mathematics skill but be able to understand the meaning of and reason for a procedure in mathematics. It is imperative that students are able to justify the reason that a mathematical statement is true (Common Core State Standards Initiative, 2021).

Additionally, the National Council of Teachers of Mathematics (NCTM) standards stipulate that instructional programs should enable all students to "monitor and reflect on the process of mathematical problem solving" (NCTM, 2023, para. 1). Engaging students in error analysis allows them to reflect upon how to accurately solve mathematics problems. Another standard described by the NCTM is that instructional programs for mathematics should enable students to "analyze and evaluate the mathematical thinking and strategies of others" (NCTM, 2023, para. 3). A significant component of elementary students participating in error analysis is understanding the mathematical thinking of others. Students must be able to explain why certain steps were taken and why they are incorrect; therefore, this communication standard is addressed through implementing error analysis into mathematics instruction.

Theory on Error Analysis in Mathematics

Students require direct instruction and guidance in the skills and strategies needed to increase understanding of mathematics concepts. Analyzing errors is a strategy that can aid in improving students' use of new, difficult fourth grade mathematics skills. Students in the fourth grade are expected to be able to comprehend and apply algorithms for solving multi-digit addition, subtraction, multiplication, and division. However, students struggle to accurately utilize computational skills beyond elementary school (Nelson & Powell, 2018). National research conducted by McKinsey and Company showed that students were at least four months behind in math at the start of the 2021-2022 school year due to past learning gaps combined with

the impacts of the COVID-19 pandemic (Sattem et al., 2022). Improved instruction in mathematics is necessary in order to improve student proficiency in mathematics (Nelson & Powell, 2018).

Historical Context

Teachers have used strategies of error analysis since the 1950s (Marzano, 2017). However, error analysis has not always been included as an instructional strategy in mathematics classrooms. For several decades, the educational pedagogy of "has relied most heavily on teachers demonstrating correctly worked example exercises as models for students to follow while practicing their own exercises. In more recent years, incorrect exercises have been introduced for the purpose of student-conducted error analysis" (Rushton, 2018, p. 1). The history of mathematics has frequently shown that educators can capitalize on errors in many ways, and failure to accomplish an initial goal has the ability to lead to revolutionary results (Borasi, 1987). Research also shows that while teachers may find value in analyzing their students' errors in mathematics, they should not be the only party involved in analyzing errors; students must be involved as well. When students are not involved, teachers and researchers are the only ones involved in looking at the error. "The students themselves would be deprived of the opportunity of engaging in the activity of attempting to 'explain' and 'fix up' their own errors – an activity that could prove to be highly motivating and challenging" (Borasi, 1987, p. 4). Historical research clearly indicates that students must be involved in error analysis just as much as teachers and researchers.

Benefits of Error Analysis

"Error analysis can create unique learning opportunities for students and should be utilized by teachers" (Mallue, 2018, p. 54). There are many benefits that are likely to accompany

the implementation of error analysis in mathematics instruction at the elementary school level. Utilizing error analysis improves students' accuracy when completing mathematics problems and improves their level of understanding of mathematics skills. In particular, error analysis inclusion in mathematics instruction can have a positive impact on brain growth and can positively impact students' mathematics comprehension.

Impact on Brain Growth

Using error analysis has positive impacts on student learning, but it also has positive effects on children's brains. Psychologist Jason S. Moser "discovered that errors contribute to brain growth. In his study, he explains that when people make errors, the brain has two potential responses: error related negativity and error positivity" (Lang-Raad & Marzano, 2019, p. 42). When children have an error-related negativity response, the brain has increased electrical activity. Then, when children become aware of the error, error positivity occurs. "Thus, making mistakes in mathematics should be an integral part of classroom activity as it not only encourages healthy motivation for learning but also contributes to deeper learning experiences through neural activity" (Lang-Raad & Marzano, 2019, p. 43). An increase in brain growth is a viable reason for teachers to implement error analysis as a strategy in their weekly mathematics instruction.

Impact on Student Accuracy

Error analysis has a positive impact upon student accuracy in mathematics. In a study conducted by Rushton (2018), students were given homework assignments, and one group was given problems that included erroneous examples. The participants in the study completed a pre-assessment prior to any homework assignments, and both groups completed a post-assessment. The Levene Statistic for the pre-assessment scores ($p > 0.05$) showed that there was not a

significant difference in the accuracy of the two groups prior to engaging in the learning process. Following the pre-assessment, only one group of students participated in error analysis. On the post-assessment, there was a significant difference in scores for the control group ((M = 5.96, SD = 4.90) and the treatment group (M = 9.41, SD = 4.77); t(51) = 2.60, p = 0.012 (Rushton, 2018). The retention of mathematical accuracy and knowledge significantly increased when error analysis was intentionally incorporated into the students' assignments. "Even though there was not a significant difference in the means, the treatment group did show a greater improvement" (Rushton, 2018, p. 6). It is clear that when students participate in error analysis, they are able to grow more in their mathematics accuracy than when they do not participate in frequent error analysis.

Impact on Student Comprehension

In addition to stimulating brain growth and positively impacting student accuracy, error analysis can positively impact students' comprehension levels. "Researchers posit a result of increased mathematical understanding when these practices are used with a combination of correctly and erroneously worked exercises" (Rushton, 2018, p. 1). Error analysis has often been utilized by teachers in order to diagnose students' misunderstandings, initiate discussions, and to clarify misconceptions. However, error analysis can also be used by students in order to increase their understanding of mathematical concepts. In fact, incorporating the instructional strategy of error analysis helps students to retain knowledge and skills in mathematics.

Error analysis for elementary students involves students being presented with a problem that has been completed incorrectly. Students then participate in analyzing and describing the error or errors that were made and also completing the problem correctly on their own. A landmark study reported that error analysis is likely to be successful when students are able to

study a specific error in an in-depth manner (Borasi, 1987). When students analyze a specific error, they can correct their own thinking processes or understand why an alternative thinking process is correct. “Error analysis leads students to enact two Standards of Mathematical Practice, namely (a) make sense of problems and persevere in solving them and (b) attend to precision” (Rushton, 2018, p. 2). Due to error analysis leading to students having an increased level of retention of knowledge, students who engage in error analysis may increase their mathematics scores on future assessments.

When error analysis is incorporated, teachers are able to observe the following behaviors in their students:

- Students actively identify and analyze their own errors.
- Students actively identify and analyze others’ errors.
- Students can describe and exemplify the different types of errors one might make.
- Students can explain how the activities have increased their understanding of the content. (Marzano, 2017, p. 42)

Research Supporting Error Analysis Implementation

There has been a great deal of research about the implementation of error analysis in mathematics classrooms. Research supports the implementation of error analysis due to its many benefits to students of all ages. While there are many examples of error analysis in research, perhaps the most renowned research about error analysis and its benefits to students of all ages has been completed by Marzano. In particular, Marzano has completed research that explains why teachers should frequently implement this instructional strategy into their mathematics instruction.

Marzano and Error Analysis

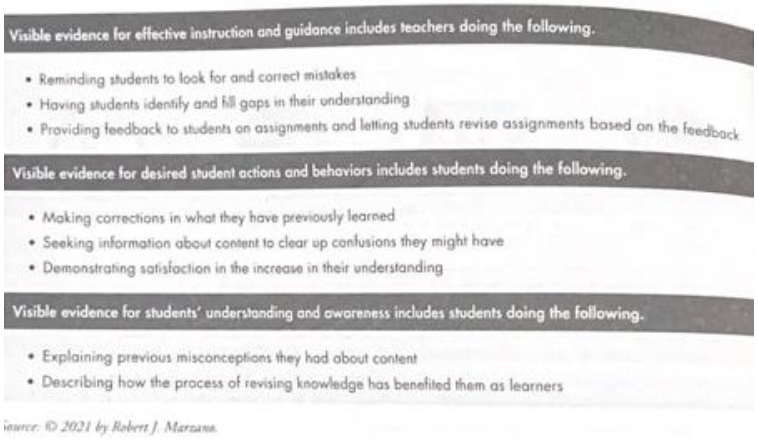
Error analysis is included in Marzano's indicators of higher order thinking skills, meaning that error analysis can be viewed as an essential instructional practice when attempting to increase students' level of understanding and thinking (Insani, et al., 2019). Students in fourth grade encounter many challenging mathematics skills and processes that are new to them and that are crucial for future mathematics success and proficiency. Due to this, it is important that teachers begin to utilize instructional strategies that work to improve students' understanding of these new, difficult concepts. Error analysis can assist students in grasping important skills, to avoid errors in their own work, and to utilize important vocabulary to describe how to solve mathematics problems. When students can explain errors in solution paths, they begin to comprehend why the correct solution path works and should be utilized when they solve future mathematics problems.

Error analysis is one of 43 key instructional elements described by Marzano and is thoroughly based upon research. Students who engage in error analysis increase their ability to use logic, which will assist them in improving their mathematics skills throughout their schooling and careers. When students examine errors, it helps to "deepen students' understanding of content by having them examine their own reasoning or the overall logic of information presented to them" (Marzano, 2017, p. 41). There are several strategies that can be utilized by teachers and students within the realm of error analysis, including identifying errors of faulty logic, identifying errors of attack, identifying errors of weak reference, identifying errors of misinformation, finding errors in the media, examining support for claims, and more (Marzano, 2017). Teachers can use multiple strategies of error analysis with their students in order to improve their understanding of mathematics content and broaden their thinking about mathematics.

Evaluating and revising instruction is a vital step when implementing a new instructional strategy. Teachers must evaluate if the new strategy is positively impacting their students, and if it is not, teachers must revise their instruction methods. In order to assess if error analysis implementation is successful for students, according to Marzano (2022), teachers should observe visible evidence in their instruction as well as in student performance. The eight examples of visual evidence for error analysis and correction described by Marzano (2022) are presented in Figure 1.

Figure 1

Visible Evidence for Examining and Correcting Errors



Source: © 2021 by Robert J. Marzano.

Marzano also suggests that teachers assess their own success and effectiveness when implementing error analysis into their instruction. Teachers should not only evaluate behaviors in their students to assess for success of a new strategy; teachers should also evaluate themselves. As teachers become more comfortable using a strategy, their instruction improves and becomes more proficient. In order to be deemed successful in utilizing error analysis in mathematics, teachers must be using the strategy correctly, and students must be clearly deepening their knowledge. Proficiency scales and rubrics are often used to assess students. These methods may also be utilized by elementary school teachers when they assess their own teaching. A self-rating

scale for examining errors in reasoning as created by Lang-Raad and Marzano (2019) is presented in Figure 2.

Figure 2

Teacher Self-Rating Proficiency Scale for Examining Errors in Reasoning

Score	Description
4: Innovating	I adapt behaviors and create new strategies for unique student needs and situations.
3: Applying	When content is informational, I engage students in activities that require them to examine their own reasoning or the logic of information as presented to them, and I monitor the extent to which students are deepening their knowledge.
2: Developing	When content is informational, I engage students in activities that require them to examine their own reasoning or the logic of information as presented to them, but I do not monitor the effect on students.
1: Beginning	I use the strategies and behaviors associated with this element incorrectly or with parts missing.
0: Not Using	I am unaware of strategies and behaviors associated with this element.

Planning for Implementation of Error Analysis

"Before students can recognize these errors, they must understand them" (Marzano, 2017, p. 44). When implementing error analysis within mathematics instruction in elementary classrooms, teachers must be intentional in their planning of their direct instruction content and strategies. "To plan, a teacher must consider the specific types of content that are the focus of instruction. If content is procedural, then structured practice is a necessity" (Marzano, 2017, p. 44). In other words, teachers must teach error analysis skills in addition to the mathematics procedures and skills directly to students before expecting students to complete the analysis on their own. Teachers must also instruct students about the new set of skills (error analysis) in a way that includes information, such as errors of attack, faulty logic, and misinformation (Marzano, 2017, p. 43). When implementing error analysis into instruction in an elementary mathematics classroom, teachers may choose to have students correct content errors in the context of a cumulative review or as an insertion into a portion of a class period (Marzano, 2022).

A possible error analysis activity presented by various former studies is to require students to be given a copy of an activity that has been completed incorrectly, correct the included mistakes, and explain the possible reasons that the person who completed the activity may have made the errors. Students correcting mistakes that were not made by themselves allows them to analyze others' errors in thinking. It also "allows students to feel more comfortable making mistakes in class – they know they are not the only ones who do it" (Marzano, 2022, p. 68).

Instructional Strategies for Error Analysis

There are many possible ways in which teachers can implement error analysis into their mathematics instruction. Marzano has presented several possible strategies that can be used for error analysis with young students. Specific instructional strategies that support elementary school teachers in the implementation of error analysis into their mathematics instruction include the following: reasoning aloud, grouping patterns in errors, and peer reviewing.

Reasoning Aloud

Reasoning aloud is a viable strategy option for teachers to utilize in their instruction. Reasoning aloud is comparable to a think-aloud, which involves students explaining their thinking or reasoning out loud to the teacher and their peers. When using this strategy, a teacher should reason out loud with their students while solving a mathematics problem and purposefully include natural errors in their thought processes (Lang-Raad & Marzano, 2019). In the end, the class will correctly solve the mathematics problem as a whole group. The goal of reasoning aloud is for students to begin to notice errors that the teacher makes. When students are able to point out errors, they become more accurate when participating in the mathematics procedure on their own. This instructional strategy also aids in helping students to understand that all people

make mistakes, even their teacher. This gives students confidence when completing their own independent work, which can also lead to increased learning and comprehension.

Grouping Patterns in Errors

Grouping patterns in errors that are made can also help students to better comprehend new mathematics content. This strategy requires students to first, solve a mathematics problem on their own. Then, the teacher will collect various students' responses. Following this, the students as a whole group will agree on the correct solution for the problem through making groups of solutions based upon certain qualities. Once the students agree upon a solution and valid solution paths, the teacher should help students to discuss consistent errors in particular strategies used by the class members (Lang-Raad & Marzano, 2019).

Peer Reviewing

Finally, teachers may choose to utilize peer reviewing as a way to implement the error analysis strategy in their classroom. This instructional strategy requires teachers to require their students to exchange their solutions with their peers. The peers then check their partner's response for errors in reasoning and mistakes. Students should talk about how they solved the problem and explain why they made certain choices when solving the problem using vocabulary that is related to the solution path. This strategy is capable of increasing students' ability to talk about their thinking and to understand why errors occurred or why a particular solution path makes the most sense (Lang-Raad & Marzano, 2019).

Error Analysis and Reluctant Learners

Students engaging in error analysis can be an asset for addressing reluctant learners in elementary classrooms. This is because error analysis assists students in understanding that mistakes help them learn. Due to this, error analysis can be a great help when students are

reluctant or nervous to learn. Specifically, “elaborative interrogation involves asking students how they know that their answer is true…[it] steers students to identify and correct their own errors which is possibly one of the most powerful things students can do to enhance their own learning” (Marzano, 2022, 135-136). Making errors, analyzing errors, and correcting errors is a key part of the learning process. Doing this consistently in a classroom will continually help students to feel comfortable during mathematics instruction, and it will help them to feel more comfortable and confident when they are completing work on their own. When students are comfortable, they are able to learn and retain more content. Making mistakes is a part of learning. “Teachers should overtly point this out, perhaps by describing times when they, too, misunderstood content and how their understanding was enhanced when they looked for the misconceptions in their own thinking” (Marzano, 2022, p. 136). Doing so will show students that mistakes are acceptable, and, in fact, that mistakes are capable of helping them grow into better learners and mathematicians.

Summary

There is no denying that intentional, research-based mathematics instructional strategies are an essential part of improving mathematics growth for students at all grade levels. The instructional strategy of error analysis is a viable option for teachers who hope to increase their students’ mathematics proficiency. It has been shown that when students are able to analyze errors in reasoning and solution paths, they find later success in mathematics accuracy and comprehension. Researchers and educators have found valuable ways to implement error analysis into mathematics instruction and have created various strategies for students to analyze errors in mathematics classrooms. With careful, intentional instruction and practice, students are able to learn and retain the mathematics skills and knowledge that they need in order to be

successful in the future. The research that has been completed has all come to the same conclusion from different parts of the world and from different grade levels: that error analysis is an advantageous strategy that improves students' mathematics proficiency and should be utilized in more classrooms. It is vital that educators begin to implement this strategy more frequently in order to improve students' understanding of mathematics content. A number of different strategies have been tested for implementing error analysis in mathematics instruction, but this has never been completed on a larger scale. Therefore, it is possible for there to be doubt that there has not been a very large-scale study done to prove beyond reasonable doubt that one specific strategy is more practical than another. Additionally, there has not been research about how much error analysis should be incorporated into mathematics instruction. While there is abundant research about the benefits if including error analysis in mathematics instruction, research does not tell if this should be completed only when students are making errors, or if it should be included in order to improve upon students' general understanding of a mathematics procedure. More research is needed in order to provide more clarity about implementing this strategy into mathematics instruction.

The research that was analyzed in this paper has outlined the benefits of error analysis, including brain growth and increased student comprehension levels. When children make errors, an error-related negativity response occurs, which stimulates increased electrical activity. Then, when students understand the error that has been made, they have error positivity, which leads to increased understanding of the skill on which they are working (Lang-Raad & Marzano, 2019). Error analysis also increases students' comprehension levels in that it can lead to better retention of content. Including error analysis activities in mathematics can help students to identify and

analyze errors, describe and exemplify different types of errors, and explain how error analysis has increased their understanding of the mathematics skill (Marzano, 2017).

The research that was analyzed also described some of the most viable strategies for error analysis to consider when planning to implement this notable strategy into mathematics instruction. Marzano's error analysis instructional strategies, such as reasoning aloud, grouping patterns in errors, and peer reviewing are approaches that have been proven to positively impact student learning. Students may analyze their own errors or others' errors in order improve their accuracy and comprehension of mathematics skills. Hopefully, in the near future, the body of research about error analysis in mathematics instruction will widen, and educators will no longer have a need to question which strategy is best when implementing error analysis or what frequency of error analysis is most advantageous for students' learning.

Methods

This study utilized a quantitative approach using quasi-experimental design. It consisted of one group of participants. There were pre-assessments and post-assessments prior to and following the intervention for the mathematics content. Additionally, there were formative assessments within the unit to check for participants' understanding and growth in mathematics. Formative assessments included exit slips and teacher-created quizzes. This enhanced participants' understanding of the mathematics content and participants' knowledge of using error analysis throughout the intervention. During the study, participants received six weeks of instruction from the researcher about how to adequately describe errors in mathematics processes. The intervention of error analysis aimed to help participants to more effectively understand how to complete mathematics problems correctly on their own. Participants were formerly introduced to writing about how they solved problems using the previously taught

mathematics skill of place value, content area exit slips, and the McGraw Hill My Math textbook prior to the start of the study.

Participants

The participants of this study were purposely selected from the teacher researcher's fourth grade general education class in East Central, Illinois. The sample included male and female participants ranging in age from nine to 10 years old. Seventeen fourth graders participated in the study. Of these participants, nine were male and eight were female. Additionally, 15 participants were White, and two participants were Black. One participant had a learning disability and had an Individualized Education Plan that stipulated that they be pulled out by a special education teacher for 30 minutes daily during mathematics and 30 minutes daily during reading. Participants in the class ranged in socioeconomic status, reading ability levels, and mathematics ability levels. Particularly, on the Northwest Evaluation Association Measures of Academic Progress (NWEA MAP) Growth Assessment taken by the student participants in August 2023, 10 scored below the 60th percentile in reading, and 11 scored below the 60th percentile in mathematics. The participant academic results are presented in Table 1.

Table 1

Number of Participants according to Designation and Category of Mathematics. n=17

	Category of Mathematics (Percentile)					
	>80th	61-80th	41-60th	21-40th	<21th	Total
Number of Participants	2	2	2	7	4	17

Note: Percentile results are based upon Fall 2023 MAP scores.

Setting

The location of this study will be in a fourth grade classroom in East Central, Illinois. The city has a population of over 16,500 people, according to the United States Census Bureau (2022). The school consists of six school buildings; there is one preschool, two elementary schools with students ranging from kindergarten to fifth grade, one middle school with students ranging from sixth to eighth grade, one high school, and LIFT (Leaders Innovating for Tomorrow), which is a regional innovation and technology center available to high school juniors and seniors in central Illinois. According to the 2022 Illinois Report Card, the school's population of 614 students includes 81.6 % White students, 2.8% Black students, 6.7% Hispanic students, and 8.8% mixed race students (Illinois State Board of Education, 2023). The school includes 326 male students and 288 female students, 66.8% of students are described as low-income students, and 18% of students have Individualized Education Plans (Illinois State Board of Education, 2023). Of the students in the school, 20.4% met or exceeded standards on the Illinois Assessment of Readiness (IAR) in English Language Arts, and 8.1% of students met standards on the IAR in Mathematics (Illinois State Board of Education, 2023).

Data Source and Research Materials

The teacher researcher utilized five main instruments to conduct this study. Participants also used handouts from the McGraw Hill My Math curriculum as they participated in activities and completed mathematics problems, including addition, subtraction, and multiplication throughout the units. The main data gathering instruments were as follows: pre-assessment and post-assessment, McGraw Hill My Math student workbooks and presentation materials, exit slips, teacher-created independent practice quizzes, and error analysis activities.

There were four mathematics units included in this study: addition and subtraction, multiplicative comparison, multiplication by a one-digit number, and multiplication by a two-

digit number. Each unit took between seven and 10 days to complete, with the exception of the multiplicative comparison unit. The multiplicative comparison unit took two days due to its lower level of complexity. All scores for the assessments, exit slips, and quizzes were recorded at the time of completion. All assessments and quizzes were developed by the teacher researcher. Concepts and sample questions for the exit slips were drawn from the McGraw Hill My Math textbook.

Pre- and- Post Assessment

The mathematics pre- and post-assessment included all skills that are incorporated in this unit: addition, subtraction, and multiplication (See Appendix A). The assessments were identical so that the participants' scores could be compared. The assessments included computation and word problems. The pre-assessment was completed by participants on the first day of the study, and the post-assessments was completed by participants on the final day of the study. These assessments showed the teacher researcher the difference in participants' accuracy prior to and following the unit of study, and it gave continuous data about participants' level of growth from the beginning to end of the mathematics unit. In addition to the mathematics problems, there was a place for participants to notate their level of understanding following the completion of the assessment. This data source was used to answer both research questions because the participants' percent of questions correct will check for accuracy, and their notation of their level of understanding describes their comprehension level.

McGraw Hill My Math Presentation and Student Workbook Materials

The McGraw Hill My Math presentation and student workbook materials were used mainly during instruction by the teacher-researcher to informally check for participants' understanding (See Appendix B). Additionally, the independent practice problems were used as

quizzes to check for accuracy when solving mathematics problems. The continuous data from the independent practice pages in the student workbook was used to answer the question, "Does the use of error analysis affect participants' accuracy when completing mathematics problems?"

Exit Slips

Exit slips were used following each mathematics lesson and following each error analysis activity (See Appendix C). The two scores were compared to evaluate if an increase occurred. Exit slips included two to three mathematics problems. Responses to exit slips were scored and assigned a percent correct. This continuous data was used to answer the research question about whether or not implementation of error analysis increases participants' accuracy when solving mathematics problems. On each exit slip, participants notated their level of understanding to show their perception of how they did on the exit slip. This answered the research question about if participants' comprehension is increased following error analysis.

Teacher-Created Independent Practice Quizzes

Teacher-created independent practice quizzes were utilized throughout the mathematics unit as formative assessments (See Appendix D). These quizzes included computation and word problems. The independent practice quizzes gave specific, continuous data about participants' accuracy when completing mathematics problems, providing data to answer the second research question. The quizzes were scored by the teacher researcher, and each participant earned a percent correct.

Error Analysis Activities

Error analysis activities (See Appendix G) for addition, subtraction, and multiplication were utilized following the initial practice exit slips. Following the initial teaching of the skill and exit slip, the participants engaged in error analysis. These activities were based upon

common mistakes made by participants. The continuous data compiled from this source included whether or not the participant correctly completed the error analysis activity. The effects of this instrument led to answered to both the first and second research questions.

Procedures of Data Collection

There were six weeks of data collection in this study. The first day of the study involved participants taking the mathematics pre-assessment, which consisted of addition, subtraction, and multiplication skills (See Appendix A). The pre-assessment was administered through a traditional paper-and-pencil assessment and consisted of computations and word problems. Participants also notated their level of understanding following the assessment.

Week One

The first unit of the study lasted 10 days and focused upon addition and subtraction skills. Included skills were adding multi-digit numbers, subtracting multi-digit numbers, and subtracting across zeros. Each skill took three days to complete. On the first day, the teacher researcher taught a mini-lesson about adding multi-digit whole numbers. Participants partook in guided practice during the mini-lesson; this guided practice was provided in the online lesson presentation from chapter two of the McGraw Hill My Math fourth grade curriculum. At the conclusion of the lesson, participants completed the adding multi-digit whole numbers exit slip (See Appendix C). These exit slips included a section in which participants notate their level of understanding. These exit slips were scored upon completion for accuracy and level of understanding.

The following day (day 3 of the study), all participants engaged in teacher-led error analysis activities focused upon adding multi-digit whole numbers. The activities focused on participants identifying the error that occurred and describing the possible reasons for the error

occurring. Participants then completed their own error analysis using a pre-solved addition problem that utilized an incorrect solution path (See Appendix G). A sum of 723,651 was given for the addition problem, 519,563 + 214,198. Participants were required to correctly solve the addition problem and explain the reason that the pre-solved problem was incorrect.

On day four of the study, the teacher researcher and participants reviewed the skill of adding multi-digit whole numbers. This was done using white boards and sample questions from chapter two of the McGraw Hill My Math fourth grade online curriculum (See Appendix B). This procedure involved the teacher researcher writing a problem for all participants to see, participants solving the problem independently, checking their solution path with a peer, and the whole group reviewing the solution together. Approximately 15 problems were completed by all participants using this procedure. At the end of the mathematics period, participants completed the adding multi-digit numbers exit slip (See Appendix C).

On day five of the study, there was a teacher-led mini-lesson about subtracting multi-digit whole numbers. The teacher researcher utilized the McGraw Hill My Math presentation materials to present this skill, and the teacher researcher along with the participants utilized the student workbook guided practice problems to practice as a whole group. Following the lesson, all participants completed guided practice and an exit slip about the topic to check for accuracy.

Week Two

The schedule for the lesson of adding multi-digit whole numbers was repeated for the topics of subtraction multi-digit numbers and subtracting across zeros. For each of these topics, there was one day of a teacher-led mini-lesson and completion of guided practice and an exit slip Then, there was one day of error analysis for each topic. Finally, the whole group reviewed the skill using white boards and then completed a final exit slip for the skill.

Specifically, on day six of the study, participants completed an error analysis activity about subtracting multi-digit whole numbers. The lesson of the day began by participants engaging in teacher-led error analysis activities focused upon subtracting multi-digit whole numbers. The activities focused on participants identifying the error that occurred and describing the possible reasons for the error occurring. Participants then completed their own error analysis using a pre-solved subtraction problem that utilized an incorrect solution path (See Appendix G). A difference of 108,778 was given for the subtraction problem, 197,053 – 88,285. Participants were required to correctly solve the subtraction problem and explain the reason that the pre-solved problem was incorrect.

On day seven of the study, a second mini-lesson to review subtracting multi-digit whole numbers was presented by the teacher-researcher using the presentation materials produced by McGraw Hill My Math. This was done using white boards and sample questions from chapter two of the fourth-grade online curriculum (See Appendix B). This procedure involved the teacher researcher writing a subtraction problem for all participants to see, participants solving the problem independently, checking their solution path with a peer, and the whole group reviewing the solution together. Approximately 15 problems were completed by all participants using this procedure. At the end of the mathematics period, participants completed the subtracting multi-digit numbers exit slip (See Appendix C).

On day eight of the study, there was a teacher-led mini-lesson about subtracting across zeros. The teacher researcher utilized the McGraw Hill My Math presentation materials to present this skill, and the teacher researcher, along with the participants, utilized the student workbook guided practice problems to practice as a whole group. Following the lesson, all participants completed guided practice and an exit slip about the topic to check for accuracy.

On day nine of the study, participants completed an error analysis activity about subtracting across zeros. The lesson of the day began by participants engaging in teacher-led error analysis activities focused upon subtracting multi-digit whole numbers. The activities focused on participants identifying the error that occurred and describing the possible reasons for the error occurring. Participants then completed their own error analysis using a pre-solved subtraction problem that utilized an incorrect solution path. A difference of 432,112 was given for the subtraction problem, 508,200 – 136,118. Participants were required to correctly solve the subtraction problem and explain the reason that the pre-solved problem was incorrect.

On day ten of the study, a mini-lesson to review subtracting multi-digit whole numbers across zeros was presented by the teacher-researcher using the materials created by McGraw Hill My Math. This was done using white boards and sample questions from chapter two of the fourth-grade online curriculum. This procedure involved the teacher researcher writing a subtraction problem for all participants to see, participants solving the problem independently, checking their solution path with a peer, and the whole group reviewing the solution together. Approximately 15 problems were completed by all participants using this procedure. At the end of the mathematics period, participants completed the subtracting across zeros exit slip.

Week Three

On day 11 of the study, participants completed a teacher-created independent practice, paper-and-pencil quiz, Adding and Subtracting Multi-Digit Whole Numbers (See Appendix D) in which they used their knowledge about addition and subtraction to solve mathematics computation and word problems.

The second unit took two days, which satisfied days 12 and 13 of the study. This unit followed a similar structure as the first unit; however, there was a slight difference in the

schedule of this unit due to its simplicity for fourth grade students. On the first day of this unit and day 12 of the study, the teacher researcher taught a mini-lesson about multiplicative comparison. Participants partook in guided practice during the mini-lesson; this guided practice was provided in the online lesson presentation from chapter three of the McGraw Hill My Math fourth grade curriculum. At the conclusion of the lesson, participants completed the multiplicative comparison exit slip (See Appendix C). These exit slips included a section in which participants notate their level of understanding. These exit slips were scored upon completion for accuracy and level of understanding.

On the second day of this unit and day 13 of the study, all participants engaged in teacher-led error analysis activities focused upon multiplicative comparison. The activities focused on participants identifying the error that occurred and describing the possible reasons for the error occurring. Participants then completed their own error analysis using a pre-solved multiplicative comparison problem that utilized an incorrect solution path (See Appendix E). A multiplication sentence of 18 x 6 = 3 was given for the multiplicative comparison problem, write eighteen is six times as many as three in equation format. Participants were required to correctly solve the multiplicative comparison problem and explain the reason that the pre-solved problem was incorrect. The final review of this skill and following exit slip were completed in conjunction with the beginning of the third unit, Multiplication by a One-Digit Number.

The third mathematics unit, Multiplication by a One-Digit Number, took seven days and satisfied days 14 through 20 of the study. This unit also followed a similar structure to the preceding units. On day 14 of the study, the teacher researcher first reviewed multiplicative comparison with the participants for approximately five minutes. This review required participants to answer multiplicative comparison questions independently on a white board.

Following this brief review, the teacher researcher taught a mini-lesson about multiplying by a one-digit number. Participants partook in guided practice during the mini-lesson, which was provided in the online lesson presentation from chapter four of the McGraw Hill My Math fourth grade curriculum. At the conclusion of the lesson, participants completed two exit slips: the multiplicative comparison exit slip as well as the multiplication by a one-digit number exit slip (See Appendix C). These exit slips included a section in which participants notate their level of understanding. These exit slips were scored upon participant completion. The exit slips were scored for accuracy and level of understanding.

One the second day of this unit and day 15 of the study, participants completed error analysis activities that were focused on the skill of multiplying by one-digit numbers. The error analysis activities required participants to identify the error that occurred and to explain the possible reasons for the error occurring. Participants then completed their own error analysis using a pre-solved addition problem that utilized an incorrect solution path (See Appendix C). A product of 410 was given for the multiplication problem, 42 x 5. Participants were expected to correctly solve the multiplication problem and explain the reason that the pre-solved problem was incorrect.

Week Four

On day three of the multiplying by a one-digit number unit and day 16 of the study, the teacher researcher and participants reviewed the skill of multiplication. This was done using white boards and sample questions from chapter four of the McGraw Hill My Math fourth grade online curriculum. This process involved the teacher researcher writing a problem for all participants to see, participants solving the problem independently, checking their solution path with a peer, and the whole group reviewing the solution together. Approximately 10 problems

were completed by all participants using this procedure. At the end of the mathematics period, participants completed the multiplying by a one-digit number exit slip (See Appendix C).

Day 17 of the study focused on the skill of multiplying multi-digit numbers. This consists of multiplying three- and four-digit numbers by a one-digit number. During this day's lesson, the teacher-researcher taught a mini-lesson about the skill. The class participated in guided practice following the mini-lesson, which was provided using the online lesson presentation from chapter four of the McGraw Hill My Math fourth grade curriculum. At the end of the lesson, participants completed the multi-digit multiplication exit slip (See Appendix C). The exit slip included a section in which participants show their level of understanding. These exit slips were scored upon participant completion. Specifically, the exit slips were scored for accuracy and level of understanding.

Day 18 of the study participants in error analysis activities that utilized the skill, multi-digit multiplication. These error analysis activities were similar to the error analysis activities completed for multiplying by a one-digit number. The desired outcome was the same: for participants to successfully be able to correctly solve the mathematics problem and explain the reason that the pre-solved solution path was incorrect. The only difference between these two error analyses was that this day required participants to analyze and solve multi-digit multiplication problems. Specifically, in this error analysis activity, a product of 8,324,048 was given for the multiplication problem, 1,456 x 8. Participants were expected to identify the error and solve the problem correctly.

Day 19 of the study mirrored the activities from the third day of the unit, with the only difference being the skill of focus. The whole class engaged in review of multiplying a multi-digit number by a one-digit number. Similar to past review days, this was accomplished using

white boards and approximately 10 sample questions from chapter four of the McGraw Hill My Math fourth grade online curriculum. At the end of the mathematics period, participants completed the multiplying multi-digit numbers exit slip (See Appendix C).

Multiplication of a multi-digit number by a one-digit number is a more complex concept for fourth grade students than previously taught topics; therefore, the teacher researcher included a second day of error analysis for this unit. On day 20, participants the teacher researcher implemented error analysis activities for participants to complete. The error analysis included the skill of multiplying multi-digit numbers. The end result was participants independently completing an error analysis of an incorrectly solved multiplication problem. A product of 7,154 was given for the multiplication problem, 1,352 x 7. Participants were expected to identify the error and solve the problem correctly.

Week Five

The final mathematics unit of this study lasted nine days and satisfied days 21 through 29 of the study. The skill taught during this unit was multiplying two-digit numbers by two-digit numbers. On day one of this unit and day 21 of the study, the teacher researcher taught the whole group a mini-lesson about the new multiplication skill. In particular, participants learned two procedures for solving complex multiplication problems: the standard algorithm and the partial products method. The materials for this mini-lesson were provided by the online lesson presentation from chapter five of the McGraw Hill My Math curriculum. Following the completion of the mini-lesson, the teacher researcher guided participants in approximately five minutes of practice with the new skill. This guided practice involved the teacher researcher and participants solving two-digit by two-digit multiplication problems together. At the end of the lesson, participants completed the two-digit multiplication exit slip (See Appendix C)**.**

On the next day of this unit and day 22 of the study, error analysis was implemented. The error analysis activities on this day were similar to the error analysis activities completed in past daily lessons. The desired outcome was that participants would be capable of to accurately solving the multiplication problem and define the reason that the pre-solved solution path was incorrect. A product of 6,032 was given for the multiplication problem, 84 x 73. Participants were expected to identify the error and solve the problem correctly.

Day 23 of the study involved the whole group reviewing the procedure for multiplying two-by-two digit numbers. This was fulfilled with individual participant white boards and sample questions from chapter five of the McGraw Hill My Math fourth grade online curriculum. This procedure involved the teacher researcher writing a problem for all students to see, participants solving the problem independently, checking their solution path with a peer, and the whole group reviewing the solution together. Approximately 8 problems were completed by all participants using this procedure. At the end of the mathematics period, participants completed the adding multi-digit numbers exit slip (See Appendix C). The number of problems included in this review is less than previous review lessons due to the amount of time necessary to solve these more intricate multiplication problems.

In order to check for understanding of the fourth grade participants, day 24 of the study required them to complete a quiz about multiplication of two-digit numbers (See Appendix E). This independent practice quiz included computation problems only in order to assess if participants could accurately use the standard algorithm or partial products method for solving multiplication problems, both of which were discussed on the first day of the unit. The scores of these quizzes were scored for accuracy and participants' level of understanding.

Day 25 of the study incorporated error analysis. Using errors made by participants on the independent practice quiz, the teacher researcher guided participants in identifying errors and debating and reasoning why these errors may have occurred. Following this, the participants were asked to independently analyze an error in a two-by-two digit multiplication problem. Participants were required to accurately solve the multiplication problem, 56 x 45, and then write an explanation for why the original solution path, with an answer of 504, was inaccurate and provide a reason the error possibly occurred.

Week Six

The sixth day of this multiplication unit and day 26 of the study involved an additional teacher-created independent practice quiz (See Appendix F). Another quiz was included due to the complexity of this multiplication process. The independent practice quiz allowed participants to further explore their knowledge of the multiplication process and allowed the teacher to check for participants' proficiency. This quiz included computational problems, and it was scored by the teacher researcher for accuracy and level of understanding.

The next two days of the multiplication by two-digit numbers unit consisted of error analysis activities. This satisfied days 27 and 28 of the study. These error analysis days followed the design of previous error analysis days and were derived from mistakes that participants had made. On the final day of the unit and day 29 of the study, participants completed another independent practice quiz (See Appendix G), which included both computation and word problems. This activity was scored immediately by the teacher researcher for the participants' accuracy and level of understanding.

On day 30 of the study, participants took the mathematics post-assessment (Appendix A). This assessment was identical to the pre-assessment. The assessment was be administered

through a traditional paper-and-pencil assessment and included computations and word problems. Participants notated their level of understanding following the assessment. The difference in score and participants' notation of their level of understanding was used to show participants' growth.

Data Analysis and Results

Data was analyzed quantitatively using descriptive statistics. The researcher collected pre-assessment and post-assessment scores for all mathematics topics: addition, subtraction, and multiplication. Additionally, the researcher collected mathematics exit slip scores for accuracy and comprehension for six weeks. During these six weeks, the weekly data collected was the result of exit slips consisting of two computation problems. Participants completed an exit slip prior to engaging in error analysis as well as an exit slip following the intervention of error analysis.

Data Analysis

The researcher used descriptive statistics to analyze quantitative data. A pre-assessment and post-assessment were given on the first and last day of the study. Each week, the researcher collected data from exit slips prior to and following participants engaging in error analysis. All of the data collected from the assessments and exit slips was organized and reported as scores using Microsoft Excel. All of the data from the study utilized central tendency, using mean and standard deviation, to answer the research questions. The analysis of data is reported in five tables. The first table was created to compare all participants' pre- and post-assessment scores as well as the difference between the two mean scores. A second table shows accuracy data from all participants' scores from the addition and one-digit multiplication exit slips prior to error analysis. A third table shows accuracy data from all participants' scores from the addition and

one-digit multiplication exit slips post error analysis. A fourth table shows comprehension level data from all participants' scores from the subtraction and two-digit multiplication exit slips prior to error analysis. A fifth table shows comprehension level data from all participants' scores from the subtraction and two-digit multiplication exit slips post error analysis.

It was hypothesized that engaging in error analysis would improve participants' ability to accurately solve addition, subtraction, and multiplication problems. The study also hypothesized that engaging in error analysis would improve participants' ability to comprehend mathematical procedures and skills. As a result, the researcher created a table showing the mean pre- and post-assessment scores and the difference between each score of all participants. Following is a discussion of the results of the study based on the research questions, does the use of error analysis affect participants' comprehension of mathematics procedures and skills, and does the use of error analysis affect participants' accuracy when completing mathematics problems?

Results

Overall results from pre- and post-assessments revealed that all participants made accuracy and comprehension gains following engagement in error analysis. This appears that error analysis had a positive effect on participants' mathematics performance. The mean pre-assessment score was 4.29 (*SD* =2.52) and the mean post-assessment score was 14.76 (*SD*= 1.55). These assessments were given on the first day and last day of the study. Participants made an overall mean score gain of 10.47 (*SD*=2.12) from the beginning to the end of the study. Table 2 below reports participants' scores from the pre- and post-assessments and the difference between the two assessments.

Table 2

Participants' Scores from Pre-Assessment and Post-Assessment. (n=17)

Participant	Pre-Assessment Score	Post-Assessment Score	Difference
1	4	16	12
2	1	12	11
3	9	16	7
4	3	15	12
5	4	16	12
6	6	13	7
7	3	15	12
8	6	15	9
9	1	16	15
10	8	16	8
11	5	15	10
12	5	16	11
13	1	13	12
14	0	11	11
15	4	16	12
16	7	16	9
17	6	14	8
Mean	4.29	14.76	10.47
Standard Deviation	2.52	1.55	2.12

Note: The items for the pre-assessment and post-assessment were identical and consisted of 16 questions.

The highest pre-assessment score was 9, and the lowest score was 0. Only one participant, participant 14, scored a 0, and this participant has an individualized education plan (IEP) for mathematics and normally receives support from a special education teacher. On this pre-assessment, the student completed the work on their own.

For the post-assessment, the highest post-assessment score was 16. The participants who scored 16 were participant 1 3, 5, 9, 10, 12, 15 and 16. The lowest-post-assessment score was 11, and participant 14 was the only one who scored the lowest. This was the same participant that scored the lowest on the pre-assessment. The participants who scored the lowest, however, made

greater growth than those who scored the highest. See Table 3 to view data of the highest and lowest achieving participants in the study.

Table 3

The Pre- and Post-Assessment of Three Highest and Three Lowest Scores of Participants. (n=6)

Participant	Pre-Assessment	Post-Assessment	Difference
Highest			
3	9	16	7
10	8	16	8
16	7	16	9
Lowest			
2	1	12	11
13	1	13	12
14	0	11	11

The research questions that drove this study were focused upon mathematics accuracy and comprehension. Therefore, it is important that data specific to these two categories is examined. The next section of results is reported based on the research questions.

Research Question 1: Does the Use of Error Analysis Affect Participants' Comprehension of Mathematical Procedures and Skills?

According to the data collected and the overall comparisons made between the exit slips before and after error analysis, participants as a whole increased their mathematics comprehension. Table 4 shows a comparison of each participant's exit slip comprehension scores prior to and following the implementation of error analysis for the mathematics skill of subtraction. This skill was analyzed to assess whether or not participants made gains in mathematics comprehension. The scores for this data were derived from participants' notation of their level of understanding. The levels of understanding were on a scale of one to four, with one

meaning that they did not understand the skill at all and four being that they could explain how to use the skill. These exit slips were completed in weeks one, two, and five.

Following the implementation of error analysis, the mean score increased by 1.82 points (*SD*=0.62) from the first exit slip to the second exit slip for subtraction. After completing both subtraction exit slips and recording the data, the total scores were compared for the first exit slip (*M*=1.65, *SD*=0.76) and the second exit slip (*M*= 3.47, *SD*=0.50). This means that the comprehension scores of participants on the second exit slip following error analysis were less varied than the scores on the first exit slip as shown by the smaller standard deviation. When looking at individual participants, all participants reached at least a level three of understanding, meaning that all participants felt that they could comprehend the subtraction tasks without assistance.

Participants 4 and 16 made the greatest growth as they went from a level one to a level four of understanding. Both of these participants scored a 1 on the first exit slip and a 4 on the second exit slip. Participants 3, 10, and 12 made the least amount of growth as their first exit slip showed the highest levels of comprehension. These participants scored a 3 on the first exit slip and a 4 on the exit slip following error analysis. The participant with an IEP, participant 14, also made the least amount of growth. This participant went from a score of 2 to 3 following error analysis with assistance from a special education teacher when completing the exit slip. See Table 4.

Table 4

Participants' Scores from Subtraction Exit Slips Prior to and Following Error Analysis. (n=17)

Comprehension Notation Subtraction Exit Slips		
Participant	Score Before Error Analysis	Score After Error Analysis

1	1	3
2	1	3
3	3	4
4	1	4
5	2	4
6	1	3
7	1	3
8	1	3
9	2	4
10	3	4
11	1	3
12	3	4
13	1	3
14	2	3
15	2	4
16	1	4
17	2	3
Mean	1.59	3.47
Standard Deviation	0.76	0.50

Note: Comprehension levels range from one to four.

Table 5 shows a comparison of each participant's exit slip comprehension scores prior to and following the implementation of error analysis for the mathematics skill of multiplication by two-digits. This skill was analyzed to assess whether or not participants made gains in mathematics comprehension. The scores for this data were derived from participants' notation of their level of understanding. Similar data was shown for the mathematics skill of multiplication by two-digits as was shown for subtraction. Following the implementation of error analysis, the mean score increased by 1.71 points (SD=0.75) from the first exit slip to the second exit slip for two-digit multiplication. After completing both multiplication by two-digits exit slips and recording the data, the total scores were compared for the first exit slip (M= 2.0, SD= 0.77) and the second exit slip (M= 3.71, SD= 0.46). This means that the comprehension levels of participants for multiplication were closer to the mean after error analysis.

When looking at individual participants, all students made growth in their comprehension levels. The three participants who made the highest growth were Participants 4, 6, and 7. These participants scored 1 on the exit slip prior to error analysis and 4 on the exit slip following error analysis. They increased their scores by 3 points. Participants 3, 7, 10, and 12 made the least amount of growth between exit slips as they initially scored the highest in comprehension on the first exit slip. Participant 14, who has an IEP, also only gained one point between exit slips. This participant also received support from a special education teacher when completing the exit slips. See Table 5.

Table 5

Participants' Scores from Multiplication by Two-Digit Exit Slips Prior to and Following Error Analysis. (n=17)

Comprehension Notation Multiplication by Two-Digit Exit Slips		
Participant	Score Before Error Analysis	Score After Error Analysis
1	2	4
2	1	3
3	3	4
4	1	4
5	2	4
6	1	4
7	1	4
8	3	4
9	2	4
10	3	4
11	1	3
12	3	4
13	2	3
14	2	3
15	3	4
16	2	4
17	2	3
Mean	2.0	3.71

Standard Deviation	0.77	0.46

Note: Comprehension levels range from one to four.

Research Question 2: Does the Use of Error Analysis Affect Participants' Accuracy When Completing Mathematics Problems?

According to the data collected and the overall comparisons made between the exit slips before and after error analysis, participants as a whole made gains in addition proficiency. Table 6 shows a comparison of each participant's exit slip scores prior to and following the implementation of error analysis for the mathematics skill of addition. This skill was analyzed to assess whether or not participants made gains in addition accuracy. Students completed exit slips, and the researcher scored each mathematics problem as either correct or incorrect. The scores students earned on these exit slips were used for this data. These exit slips were completed in week one.

Following the implementation of error analysis, the mean score increased by 0.59 points (SD=0.60) from the first exit slip to the second exit slip for addition. After completing both addition exit slips and recording the data, the total scores were compared for the first exit slip (M= 1.35, SD= 0.68) and the second exit slip (M= 1.94, SD= 0.24). This shows that the accuracy of participants for addition increased and were less varied after error analysis than prior to error analysis.

When looking at individual participants, all eight began with a perfect score of two out of two questions answered correctly remained at that level. The participants who began on the first exit slip with a score of 2 were Participants 1, 3, 6, 7, 10, 11, 12, and 17. The other nine participants made gains of one or two points in their scores following error analysis. More specifically, Participant 9 had the highest amount of growth with a score of 0 on the first exit slip

and a score of 2 on the exit slip following error analysis. Participants 1, 3, 6, 7, 10, 11, 12, and 17 made the least amount of growth due to them scoring a 2 on the first exit slip and a 2 on the second exit slip. These participants initially scored the highest, and they made the least growth as growth was not possible. The participant who has an IEP, Participant 14, went from a score of 0 to 1 on the two exit slips with the support from a special education teacher in addition to the general education teacher. See Table 6.

Table 6

Participants' Scores from Addition Exit Slips Prior to and Following Error Analysis. (n=17)

Participants' Scores on Addition Exit Slips		
Participant	Score Before Error Analysis	Score After Error Analysis
1	2	2
2	1	2
3	2	2
4	1	2
5	1	2
6	2	2
7	2	2
8	1	2
9	0	2
10	2	2
11	2	2
12	2	2
13	1	2
14	0	1
15	1	2
16	1	2
17	2	2
Mean	1.35	1.94
Standard Deviation	0.68	0.24

Note: This exit slip was out of a total of 2 points.

Tables 7 shows a comparison of each participant's exit slip scores prior to and following the implementation of error analysis for the mathematics skill of multiplication by one-digit. This skill was analyzed to assess whether or not participants made gains in multiplication accuracy. Students completed exit slips, and the researcher scored each mathematics problem as either correct or incorrect. The scores students earned on these exit slips were used for this data. These exit slips were completed in week four. According to the data collected and the overall comparisons made between the exit slips before and after error analysis, participants as a whole made gains in multiplication proficiency.

Similar data was shown for the mathematics skill of multiplication by one-digit as shown for addition. Following the implementation of error analysis, the mean score increased by 0.82 points (SD=0.71) from the first exit slip to the second exit slip for one-digit multiplication. After completing both exit slips and recording the data, the total scores were compared for the first exit slip (M= 1.33, SD= 0.68) and the second exit slip (M= 2.0, SD=0). This shows that accuracy levels of participants for multiplication by one-digit increased and were closer to the mean after error analysis.

When looking at individual participants, all six who began with a perfect score remained at that level. The other 11 participants made gains in their scores following error analysis with multiplication by one-digit. Participants 1, 2, and 5 showed the highest amount of growth. Participants 1 and 5 were absent for the first exit slip and scored a 2 on the second exit slip. Participant 2's score on the exit slip prior to error analysis was 0, and the score following error analysis was 2. The participants who made the least amount of growth between exit slips were Participants 3, 6, 10, 12, 15, and 16. These participants each scored 2 on the first exit slip and 2 on the second exit slip. The participant with an IEP, Participant 14, gained one point between

exit slips. The participant scored a 1 on the first exit slip and a 2 on the second exit slip. The student received support from a special education teacher. See Table 7.

Table 7

Participants' Scores from Multiplication by One-Digit Exit Slips Prior to and Following Error Analysis. (n=17)

Participants' Scores on Multiplication by One-Digit Exit Slips		
Participant	Score Before Error Analysis	Score After Error Analysis
1	-	2
2	0	2
3	2	2
4	1	2
5	-	2
6	2	2
7	1	2
8	1	2
9	1	2
10	2	2
11	1	2
12	2	2
13	1	2
14	1	2
15	2	2
16	2	2
17	1	2
Mean	1.33	2.0
Standard Deviation	0.59	0

Note: The symbol - represents a participant who was absent.

The results from this study showed that participants' accuracy and comprehension of addition, subtraction, and multiplication increased following partaking in error analysis.

Following participating in error analysis activities for addition, subtraction, and multiplication, participants made gains in their scores.

Findings

Based upon the data collected during the study, the strategy of error analysis was effective in increasing participants' mathematics proficiency in fourth grade. When examining the group of participants overall, regardless of mathematics achievement designations, error analysis was effective with 100 percent of participants from beginning to end of the study. Error analysis was effective in increasing comprehension scores in 100 percent of participants and increasing accuracy scores in 58 percent of participants. The participants that did not show growth in accuracy completed the exit slip prior to error analysis implementation.

The purpose of the study was to determine the effect implementing error analysis activities has on student understanding of fourth grade mathematics concepts. The research questions for this study focused on mathematics comprehension and mathematics accuracy following the implementation of error analysis activities. The study hypothesized that engaging in error analysis would improve participants' ability to accurately solve addition, subtraction, and multiplication problems. The study also hypothesized that engaging in error analysis would improve participants' ability to comprehend mathematical procedures and skills. The study concluded that 100 percent of fourth grade participants made growth or received two perfect scores when graded for accuracy, and 100 percent of fourth grade participants made growth in mathematics comprehension following error analysis. Therefore, in this study, implementing error analysis into mathematics instruction did increase fourth graders' overall mathematics proficiency in addition, subtraction, and multiplication.

Conclusion and Discussion

Conclusion

Error analysis implementation in mathematics instruction in a fourth grade classroom did prove to be effective as hypothesized by the researcher. When conducting the study, the researcher observed that the fourth grade participants became more confident in their mathematics skills after engaging in error analysis activities. This helped to increase their accuracy and comprehension levels. For example, when learning about subtraction, students began with confusion and unease about regrouping. The error analysis activity focused on errors made in regrouping. Following this, the students understood how regrouping is completed correctly and were able to successfully complete the second exit slip with no errors in regrouping. Error analysis allows students and educators to identify common problems and mistakes and focus on how to fix them. This, in turn, helps students to avoid these errors when they are completing mathematics problems on their own in the future.

Discussion

The results of the study showed that participants grew in mathematics proficiency following the implementation of error analysis. There were two research questions in this study: Does the use of error analysis affect participants' comprehension of mathematical procedures and skills, and does the use of error analysis affect participants' accuracy when completing mathematics problems? The results from the study showed that participants' comprehension and accuracy were positively affected by error analysis. The participants' scores increased following error analysis. The study hypothesized that engaging in error analysis would improve participants' comprehension when solving mathematics problems. The study also hypothesized that participants engaging in error analysis would assist them in solving future mathematics problems accurately. The results from this study proved the hypothesis was true.

Implications and Limitations

Implications

Educators of all grade levels have countless choices when it comes to instructional methods and strategies to utilize in their classrooms. Educators also have a multitude of different learning styles and academic ability levels within their students that must be taught. It is imperative that educators utilize applicable practices in their classrooms to address all learners and their needs in order to effectively promote academic growth.

Mathematics instruction is one of the most significant points of discussion in education today due to the decrease in mathematics proficiency in students at all grade levels post-pandemic. Students' mathematics scores on standardized tests have seen a decrease since this time. Because of this, it is imperative that educators choose to implement effective and engaging strategies for mathematics instruction. Extensive research has shown that error analysis can assist students in grasping important skills, to avoid errors in their own work, and to utilize important vocabulary to describe how to solve mathematics problems. When students can explain errors in solution paths, they begin to comprehend why the correct solution path works and should be utilized when they solve future mathematics problems. This increased understanding of mathematics problems and possible errors will lead to increased proficiency in mathematics and an increase in students' scores on standardized tests.

The results of this study showed that, for the participants in the fourth grade classroom used in this study, error analysis was positively effective for all levels of students when it comes to mathematics accuracy and comprehension. This is advantageous for educators in this particular school because they are looking for strategies to improve mathematics instruction. Also, educators in this particular school utilize Marzano's higher order thinking skills in

conjunction with Marzano's proficiency scales for mathematics instruction. Fourth grade educators in this school in particular can be confident that their students will increase their mathematics proficiency using the error analysis strategy. They can also be confident in supplementing their current mathematics curriculum, McGraw Hill MyMath, with error analysis activities with more frequency. The data collected in the study showed that student who scored the lowest on the pre-assessment also scored the lowest on the post-assessment. This student made the least amount of growth following error analysis. It may be appropriate for the educator to utilize additional strategies from the MyMath curriculum to aid in mathematics growth in addition to error analysis activities. This student also receives support in special education, and extra supports could be provided.

The administration of the school where the study was completed can be assured that the adjustments to mathematics instruction in the elementary grades that have been suggested for educators to use with students is effective in helping students in the fourth grade make gains in mathematics accuracy, comprehension, and proficiency as a whole. The elementary school and school district have spent time and resources on increasing educators' understanding of error analysis and Marzano's higher order thinking skills, and this study shows that it is a good use of their time and resources.

Limitations

One limitation of the study was that the sample of participants was quite small, and the study took place in a singular classroom. A larger sample of students and educators would give more reliable data and results. Additionally, conducting the study across multiple grade levels would give a more accurate account of effectiveness of error analysis upon mathematics proficiency.

A second limitation was that some participants missed a day or more of instruction during the six week study due to illness, appointments, or repeated truancy. When participants miss portions of the instruction, it is possible that results could be skewed as these participants did not receive the exact same amount of error analysis instruction as their peers in the study. Participants were only absent on days in which the class was receiving instruction about a mathematics topic, completing exit slips, completing quizzes, or completing error analysis activities. No participants were absent on the days of the pre-assessment and post-assessment.

Action Plan

The researcher plans to enlist fellow fourth grade teachers to experiment with supplementing the current McGraw Hill MyMath curriculum with more error analysis activities to see if they get similar results to this study. The researcher also plans to later enlist fellow teachers in other grade levels to examine if they get similar results to this study at different levels. Additionally, the researcher plans to present this study at a future academic team meeting to share the findings of the study with colleagues along with strategies from Marzano's higher order thinking skills to try when students struggle with mathematics accuracy and comprehension. The data and findings of the study will be presented in a professional poster at Eastern Illinois University.

The researcher suggests that more research be conducted on the effects of error analysis on mathematics instruction to determine how it can be used most effectively. It would be beneficial to conduct studies with larger participant samples, multiple grade levels, and a longer study to produce more reliable data. In the meantime, the researcher will use the knowledge gained from this study to better serve students in the classroom when it comes to mathematics instruction.

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Appendix A

Pre-Assessment and Post-Assessment

Name: _____ Level of Understanding: _____

Mathematics Assessment

Addition, Subtraction, and Multiplication

Part 1: Addition and Subtraction Skills

Complete the patterns.

1. 370,845 + ☐ = 370,945 ☐ = _____	2. 21,429 - 100 = ☐ ☐ = _____

Find the exact sum or difference.

3. 44,356 + 43,985	4. $179,259 + $157,963
5. 17,435 - 9,665	6. 941,803 - 589,756

Part 2: Multiplication Skills

Answer the following questions using the method of multiplication that works best for you.

7. 46 x 2	8. 4 x 500
9. 1,259 x 5	10. 91 x 73
11. 47 x 96	12. 40 x 15

Part 3: Word Problems

Answer the following questions using addition, subtraction, or multiplication..

13. There were 88,000 people at the concert. Of those people, 2,556 got free tickets. How many people had to pay for their tickets?

_____

14. Bob had $9,000. He spent $5,000 on a TV and $2,961 on a stereo system. How much money does he have left?

_____

15. The average person eats 57 hot dogs per year. Elsie is 82 years old. *About* how many hot dogs would she have eaten during her lifetime? (Round to the nearest ten.)

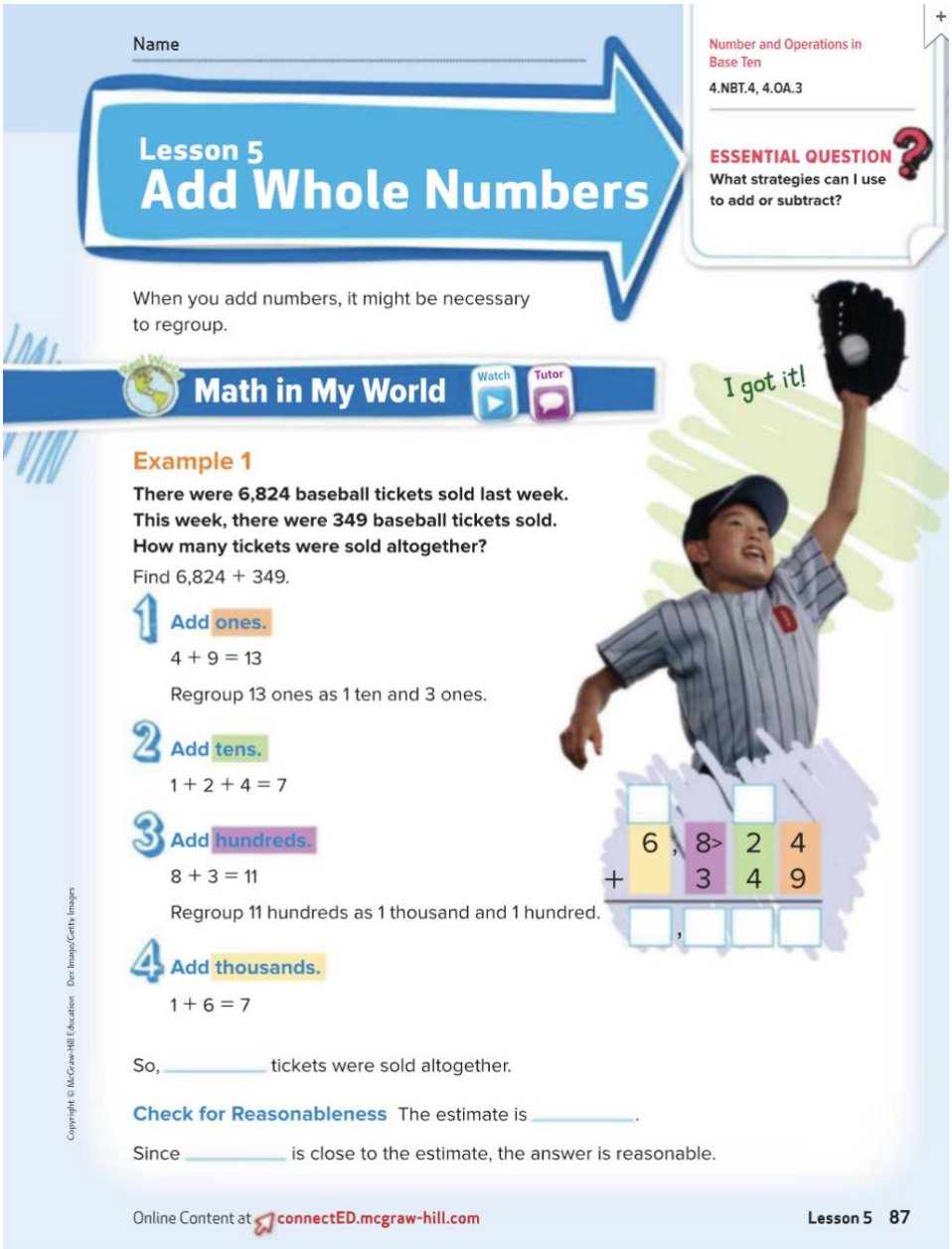
_____

16. Dr. Fisher is a history professor. He teachers 18 classes each year. He has 39 students in each class. How many total students does she teach?

_____

Appendix B

McGraw Hill My Math Materials



Name _____

Number and Operations in Base Ten

4.NBT.4, 4.OA.3

Lesson 5

Add Whole Numbers

ESSENTIAL QUESTION

What strategies can I use to add or subtract?

When you add numbers, it might be necessary to regroup.

Math in My World

Watch Tutor

I got it!

Example 1

There were 6,824 baseball tickets sold last week. This week, there were 349 baseball tickets sold. How many tickets were sold altogether?

Find 6,824 + 349.

1 **Add ones.**

4 + 9 = 13

Regroup 13 ones as 1 ten and 3 ones.

2 **Add tens.**

1 + 2 + 4 = 7

3 **Add hundreds.**

8 + 3 = 11

Regroup 11 hundreds as 1 thousand and 1 hundred.

4 **Add thousands.**

1 + 6 = 7

	6	8	2	4
+		3	4	9

So, _____ tickets were sold altogether.

Check for Reasonableness The estimate is _____.

Since _____ is close to the estimate, the answer is reasonable.

Example 2

Watch

Ticket sales for a play are shown in the table. What were the total sales?

Ticket Sales	
Day	Amount
Saturday	$58,713
Sunday	$43,827

Estimate

$58,713 rounds to $60,000
+ $43,827 rounds to + $40,000
$100,000

1 Add ones.

3 + 7 = 10

Regroup 10 ones as 1 ten and 0 ones.

2 Add tens.

1 + 1 + 2 = 4

3 Add hundreds.

7 + 8 = 15

Regroup 15 hundreds as 1 thousand and 5 hundreds.

4 Add thousands.

1 + 8 + 3 = 12

Regroup 12 thousands as 1 ten thousand and 2 thousands.

5 Add ten thousands.

1 + 5 + 4 = 10

Place the $ symbol in front of the sum.

So, the total ticket sales were _____.

Talk MATH

Explain why it is important to line up digits in numbers when you add.

Guided Practice

Add. Estimate to check your work.

1. $2,961
   + $4,205

2. 29,380
   + 10,225

Name _____

Independent Practice

Add. Estimate to check your work.

3. 8,346
   + 7,208

4. $23,824
   + $7,346

5. 82,828
   + 4,789

6. $37,178
   + $82,370

7. $693,782
   + $47,816

8. 743,980
   + 211,315

9. 254,671
   + 381,366

10. $15,789
    + $22,503

11. 56,772
    + 29,428

Add. Use the place-value chart to help set up the problem.

12. 17,599 + 72,682 = _____

	Thousands			Ones		
	hundreds	tens	ones	hundreds	tens	ones
+						

Problem Solving

13. There are 4,585 students who rode the bus to school today. There were 3,369 students who came to school another way. How many students were there in all at the school?

Brain Builders

14. Mathematical PRACTICE 6 **Explain to a Friend** Becky's mom wants to buy a new TV that costs $1,579 and a DVD player that costs $247. She has $2,000. If she buys groceries for $136, will she have enough money for the TV and the DVD player? Explain to a friend.

15. Mr. Russo's class is collecting bottles to recycle. The class collected some bottles in March and 2,555 bottles in April. 3,701 bottles were collected in all. How many were collected in March?

16. Mathematical PRACTICE 1 **Make Sense of Problems** Write two 5-digit addends whose sum would give an estimate of 60,000. Explain your reasoning.

17. **Building on the Essential Question** Explain why an addition problem that has 4-digit addends could have a 5-digit sum.

Name _____

Number and Operations in Base Ten

4.NBT.4, 4.OA.3

Lesson 6
Subtract Whole Numbers

ESSENTIAL QUESTION
What strategies can I use to add or subtract?

Subtraction of whole numbers is similar to addition of whole numbers because you might need to regroup.

Math in My World

Watch Tools Tutor

Example 1

The Trevino family is moving to a new city. They have driven 957 miles out of the 3,214 miles that they need to drive. How many more miles do they need to drive?

Find 3,214 − 957.

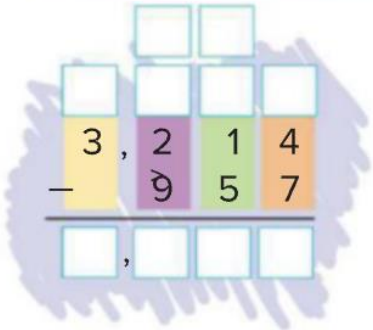
1 Subtract ones.
Regroup a ten as 10 ones.
10 ones + 4 ones = 14 ones
14 ones − 7 ones = _____ ones

2 Subtract tens.
Regroup a hundred as _____ tens.
10 tens + 0 tens = 10 tens
10 tens − 5 tens = _____ tens

3 Subtract hundreds.
Regroup a thousand as _____ hundreds.
10 hundreds + 1 hundred = 11 hundreds
11 hundreds − 9 hundreds = _____ hundreds

4 Subtract thousands.
2 thousands − 0 thousands = _____ thousands

So, 3,214 − 957 = _____. The Trevino family needs to drive _____ more miles.



The **minuend** is the first number in a subtraction sentence from which a second number is to be subtracted. The **subtrahend** is the number that is subtracted.

Example 2

The band has raised $1,345 toward new equipment. If the goal is to raise $4,275, how much money must still be raised?

Estimate $4,275 rounds to $4,300

−$1,345 rounds to $1,300

$3,000

1 Subtract ones.
5 − 5 = 0

2 Subtract tens.
7 − 4 = 3

3 Subtract hundreds.
Regroup a thousand as 10 hundreds.
12 − 3 = 9

4 Subtract thousands.
3 − 1 = 2

$ 4 , 2 7 5 ← minuend
−$ 1 , 3 4 5 ← subtrahend
$, ☐ ☐ ☐

So, the band still needs to raise _____.

Check for Reasonableness You can use addition to check your subtraction.

4,275 − 1,345 = 2,930 → 2,930 + 1,345 = 4,275

The answer is correct and close to the estimate.

Talk MATH

Explain how to check the answer to a subtraction problem by using addition.

Guided Practice

Subtract. Use addition or estimation to check.

1. 2,962 − 845

2. $4,785 − $2,293

Name _____

Independent Practice

Subtract. Use addition or estimation to check.

3. 8,845 − 627

4. $5,751 − $4,824

5. $8,327 − $5,709

6. 39,536 − 18,698

7. 847,311 − 562,530

8. 93,458 − 21,649

9. 78,215 − 56,827

10. $18,345 − $14,400

11. 629,843 − 216,954

Subtract. Use addition or estimation to check. Use the place-value chart to set up the problem.

12. 961,344 − 345,822 = _____

	Thousands			Ones		
	hundreds	tens	ones	hundreds	tens	ones
−						

13. Do you prefer to use addition or estimation to check? Explain.

_____

_____

Name _____

Number and Operations in Base Ten

4.OA.1, 4.OA.2

Lesson 3 Multiplication as Comparison

ESSENTIAL QUESTION
How are multiplication and division related?

Sometimes a problem uses a phrase like *times as many*, *times more*, and *times as much*. These kinds of problems are comparison problems.

Math in My World

Tools Watch Tutor

Example 1

Mary attended camp for 7 days this summer. Tyler attended 3 times as many days as Mary. Find the number of days Tyler attended camp.

Use counters to help you compare the groups of days.

1. Model Mary's days at camp as _____ group of 7 days. Draw your model.

2. Tyler attended 3 times as many days at camp.

   Model Tyler's days at camp as _____ groups of 7. Draw your model.

3. Find the total of 3 groups of 7.

   _____ + _____ + _____ = _____

   or

   _____ × _____ = _____

So, Tyler attended camp for _____ days.

A type of model drawing is the bar diagram. A bar diagram can help you understand a problem and plan to solve it.

Example 2

Suki used 15 beads to make a bracelet. This is 3 times as many beads as what Cassady used. How many beads did Cassady use?

1 The bar diagram models this problem.

Cassady [? beads]

15 beads

Suki [? beads] [? beads] [? beads]

Suki used _____ times as many beads as Cassady.

2 Find how many beads Cassady used. Write an equation.

$3 \times ? = 15$ $3 \times$ _____ = 15 beads ← Use a fact family.

So, Cassady used _____ beads.

Guided Practice

1. Use multiplication or division to complete the equation for the phrase below.

3 times as much

$3 \times 3 = ?$

$3 \times 3 =$ _____

Talk MATH

One way to interpret $24 = 8 \times 3$ is to say that 24 is 8 times as many as 3. What is another way you can interpret this equation?

Name _____

Independent Practice

Use multiplication or division to complete each equation and/or drawing.

2. 3 times as many

_____ × _____ = 3

3. 5 times more

_____ × _____ = 25

4. 4 times as much

_____ × 3 = 12

5. 10 times as much

10 × _____ = 40

6. 2 times more

2 × _____ = 6

7. twice as many

2 × _____ = 14

Complete each bar diagram. Then complete the multiplication equation.

8. twice as many as 4 boys

boys

2 × _____ = 8

9. 2 times as many as 3 bows

bows

_____ × _____ = _____

10. 4 times as many as 6 fish

fish

_____ × _____ = _____

Draw a bar diagram. Then write a multiplication equation.

11. 3 times as much money as $6

Write the equation.

_____

12. 5 times as many as 1 star

Write the equation.

_____

Problem Solving

Draw a bar diagram and write an equation to solve.

13. Mathematical PRACTICE 4 **Model Math** There are 3 times as many blue balloons as green balloons. There are 4 green balloons. How many blue balloons are there?

Find the unknown.

_____ is 3 times as many as 4.

14. Nan needs 4 times as much flour as sugar. She needs 4 cups of sugar. How much flour does she need?

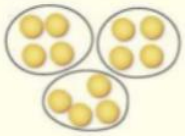
Find the unknown.

_____ is 4 times as many as 4.

Brain Builders

15. Mathematical PRACTICE 2 **Use Number Sense** Circle the example that does not belong. Explain your reasoning.

$4 + 4 + 4 = 12$



$12 - 4 = 8$

16. **Building on the Essential Question** How can a bar diagram help me plan and solve a problem? Explain.

Name _____

Lesson 5
Multiply by a Two-Digit Number

Number and Operations in Base Ten

4.NBT.5

ESSENTIAL QUESTION

How can I communicate multiplication?

Place value can help you multiply.

Math in My World

Tutor

Example 1

Ann's mom buys two helmets. Each helmet costs $24. How much does she spend on helmets?

You need to find 2 × $24.

Multiply.

1 Multiply the ones.

$$\begin{array}{r} 24 \\ \times\ 2 \\ \hline 8 \end{array}$$

2 × 4 ones = _____ ones

2 Multiply the tens.

$$\begin{array}{r} 24 \\ \times\ 2 \\ \hline 48 \end{array}$$

2 × 2 tens = _____ tens

So, Ann's mom spends _____.

Helpful Hint

Line up the factors by the ones place.

Check for Reasonableness

The area model shows the partial products.

	20	4
2	2 × 20 = 40	2 × 4 = 8

40 + 8 = ☐

So, the answer is correct.

You can estimate to check for reasonableness.

Example 2

Thirty-one bikes were ordered from Jimmy's Bike Shop. Each bike has two tires. How many tires will Jimmy need for the bikes?

Estimate $31 \times 2 \rightarrow$ ____ × ____ = ____

$$\begin{array}{r} 31 \\ \times\ 2 \\ \hline \square\square \end{array}$$

1 Multiply the ones.

2 × 1 one = 2 ones

Write the product in the ones place.

2 Multiply the tens.

2 × 3 tens = 6 tens

Write the product in the tens place.

So, Jimmy will need ____ bike tires.

Check for Reasonableness

The product, ____, is close to the estimate, ____.

Guided Practice

Multiply. Check for reasonableness.

1. $\begin{array}{r} 42 \\ \times\ 2 \\ \hline \square\square \end{array}$

2. $\begin{array}{r} 21 \\ \times\ 3 \\ \hline \square\square \end{array}$

3. $11 \times 4 =$ ____

4. $32 \times 2 =$ ____

Talk MATH

Suppose you found 99 as the product of 33 × 3. How can you check to see if your answer is reasonable?

Name ____

Independent Practice

Multiply. Check for reasonableness.

5. $\begin{array}{r} 44 \\ \times\ 2 \\ \hline \square\square \end{array}$

6. $\begin{array}{r} 21 \\ \times\ 4 \\ \hline \square\square \end{array}$

7. $\begin{array}{r} 13 \\ \times\ 2 \\ \hline \square\square \end{array}$

8. $41 \times 2 =$ ____

9. $12 \times 3 =$ ____

10. $4 \times 22 =$ ____

Algebra Find each unknown number.

11. $41 \times 2 = h$

$h =$ ____

12. $12 \times 3 = j$

$j =$ ____

13. $4 \times 22 = k$

$k =$ ____

Problem Solving

14. Mathematical PRACTICE 1 **Plan Your Solution** A city has 23 swing sets. Each has 3 swings. How many swings are there in all?

15. The world's largest rodent is called a *capybara*. It can weigh 34 kilograms. How much could 2 capybaras weigh?

Brain Builders

16. Recycling one ton of paper saves 17 trees. About how many trees are saved if 2 tons of paper are recycled the first month and 3 tons are recycled the second month?

17. Mathematical PRACTICE 6 **Explain to a Friend** Adrian has four boxes of action figures. There are 12 in each box. Alec has 21 action figures in each of his three boxes. Who has more action figures? Explain to a friend.

18. **Building on the Essential Question** How can estimation be used to check multiplication problems for reasonableness? Give an example.

Name _____

Number and Operations in Base Ten

4.NBT.5

Lesson 4
Multiply by a Two-Digit Number

ESSENTIAL QUESTION
How can I multiply by a two-digit number?

Math in My World

Watch Tutor

Example 1

A coyote travels 27 miles each hour. How far can a coyote travel in 12 hours?

Find 27 × 12.

One Way **Use partial products.**

Draw an area model. Separate each factor into tens and ones. Multiply. Then add the partial products.

	10	2
20	200	40
7	70	14

200 + 40 + 70 + 14 = _____

Another Way **Use paper and pencil.**

1 **Multiply the ones.**

$$\begin{array}{r} 27 \\ \times\ 2 \\ \hline 54 \end{array}$$

7 × 2 = 14
Regroup the tens.
2 tens × 2 ones = 4 tens
4 tens + 1 ten = 5 tens

2 **Multiply the tens.**
27 × 1 ten = 27 tens, or 270

3 **Add the products.**
54 + 270 = 324

$$\begin{array}{r} 2\ 7 \\ \times\ 1\ 2 \\ \hline \square\ \square \\ +\ \square\ \square\ \square \\ \hline \square\ \square\ \square \end{array}$$

27 × 12 = _____

So, a coyote can travel _____ miles in 12 hours.

Example 2

Heidi's monthly bills are shown. How much does she spend on her cell phone service in 2 years?

Write an equation to help you solve the problem.

$38 × 12 × 2 = m

Heidi's cell phone bill each month · 12 months in one year · 2 years

Monthly Bills	
Cable	$55
Cell phone	$38
Movie club	$21
Water	$93

You know that 12 × 2 = 24. So, you need to find $38 × 24.

Estimate _____ × _____ = _____

$ 3 8
× 2 4

1 Multiply the ones. → $ ☐☐☐

2 Multiply the tens. → + $ ☐☐☐

3 Add the products. → $ ☐☐☐

So, the cost of cell phone service for 2 years is $ _____.

Check

_____ is close to the estimate of _____.

Talk MATH

Explain the steps needed to find the product of 56 and 23.

Guided Practice

Multiply.

1.
$$\begin{array}{r} 3\ 5 \\ \times\ 2\ 4 \\ \hline \square\square\square \\ +\ \square\square\square \\ \hline \square\square\square \end{array}$$

Multiply the ones.

Multiply the tens.

Add.

	30	5
20	600	100
4	120	20

The area model shows that

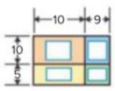
600 + 120 + 100 + 20 = _____.

Name _____

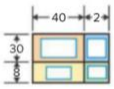
Independent Practice

Multiply. Use the area model to check.

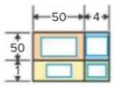
2. 19 × 15



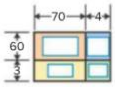
3. 42 × 38



4. $54 × 51



5. $74 × 63



Multiply.

6. 47 × 24

7. 64 × 46

8. 83 × 67 = _____

9. 91 × 78 = _____

Problem Solving

10. A greyhound dog can jump a distance of 27 feet. How many feet will a greyhound travel if it jumps 12 times?

11. The fourth-grade students at Tremont School received a ribbon if they read 50 books during the school year. The school gave out 69 ribbons at the end of the year. How many books did the students read in all?

12. Mathematical PRACTICE 2 **Use Symbols** Each day, enough paper is recycled in the United States to fill 15 miles of train boxcars. How many miles of boxcars could be filled in 5 weeks? Complete the equation to help you solve the problem.

$15 \times 5 \times$ _____ $= b$

Brain Builders

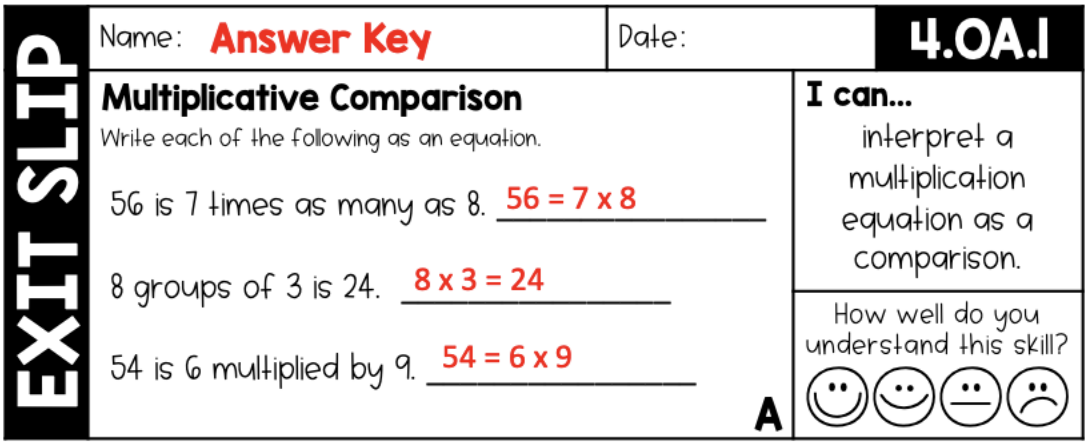
13. Mathematical PRACTICE 3 **Which One Doesn't Belong?** Circle the multiplication problem that does not belong with the other three. Explain and rewrite the problem so that it does belong.

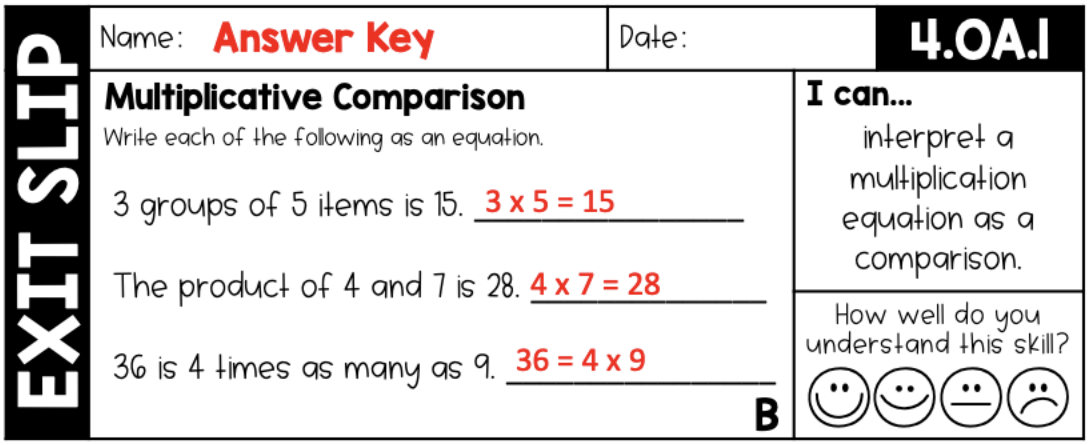
22 × 15	$45 × 28	37 × 18	$66 × 25

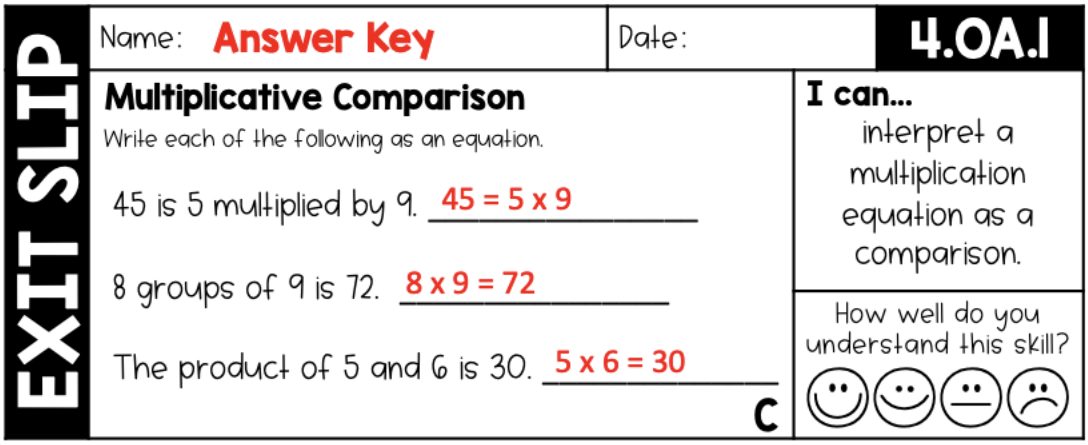
14. **Building on the Essential Question** Why can't the product of two 2-digit numbers ever be two digits? Explain.

Appendix C

Mathematics Exit Slips







EXIT SLIP

Name: **Answer Key** Date: **4.NBT.4**

Addition

Find the sum.

$$\begin{array}{r} 43{,}635 \\ +\ 22{,}594 \\ \hline 66{,}229 \end{array} \qquad \begin{array}{r} 329{,}429 \\ +\ 238{,}236 \\ \hline 567{,}665 \end{array}$$

A

I can...

add multi-digit whole numbers.

How well do you understand this skill?

EXIT SLIP

Name: **Answer Key** Date: **4.NBT.4**

Addition

Find the sum.

$$\begin{array}{r} 52{,}826 \\ +\ 34{,}287 \\ \hline 87{,}113 \end{array} \qquad \begin{array}{r} 435{,}738 \\ +\ 346{,}425 \\ \hline 782{,}163 \end{array}$$

B

I can...

add multi-digit whole numbers.

How well do you understand this skill?

EXIT SLIP

Name: **Answer Key** Date: **4.NBT.4**

Addition

Find the sum.

$$\begin{array}{r} 65{,}229 \\ +\ 36{,}374 \\ \hline 101{,}603 \end{array} \qquad \begin{array}{r} 273{,}534 \\ +\ 249{,}372 \\ \hline 522{,}906 \end{array}$$

C

I can...

add multi-digit whole numbers.

How well do you understand this skill?

EXIT SLIP

Name: **Answer Key** Date: **4.NBT.4**

Subtraction

Find the difference.

47,625	543,361
− 37,387	− 293,292
10,238	250,069

A

I can...

subtract multi-digit whole numbers.

How well do you understand this skill?

EXIT SLIP

Name: **Answer Key** Date: **4.NBT.4**

Subtraction

Find the difference.

63,594	749,609
− 48,663	− 392,763
14,931	356,846

B

I can...

subtract multi-digit whole numbers.

How well do you understand this skill?

EXIT SLIP

Name: **Answer Key** Date: **4.NBT.4**

Subtraction

Find the difference.

75,037	829,334
− 56,353	− 354,456
18,684	474,878

C

I can...

subtract multi-digit whole numbers.

How well do you understand this skill?

EXIT SLIP

Name: **Answer Key** Date: **4.NBT.5**

Multiplication By 1-Digit

Find the product.

94	329	2,466
x 8	x 6	x 4
752	1,974	9,864

A

I can... multiply a whole number up to four digits by a one-digit whole number.

How well do you understand this skill?

EXIT SLIP

Name: **Answer Key** Date: **4.NBT.5**

Multiplication By 1-Digit

Find the product.

87	535	3,720
x 5	x 7	x 4
435	3,745	14,880

B

I can... multiply a whole number up to four digits by a one-digit whole number.

How well do you understand this skill?

EXIT SLIP

Name: **Answer Key** Date: **4.NBT.5**

Multiplication By 1-Digit

Find the product.

53	726	5,038
x 7	x 8	x 6
371	5,808	30,228

C

I can... multiply a whole number up to four digits by a one-digit whole number.

How well do you understand this skill?

EXIT SLIP

Name: **Answer Key** Date: **4.NBT.5**

Multiplication By 2-Digits

Find the product.

54 x 26 = 1,404	73 x 39 = 2,847

A

I can... multiply two 2-digit numbers.

How well do you understand this skill?

EXIT SLIP

Name: **Answer Key** Date: **4.NBT.5**

Multiplication By 2-Digits

Find the product.

66 x 34 = 2,244	92 x 25 = 2,300

B

I can... multiply two 2-digit numbers.

How well do you understand this skill?

EXIT SLIP

Name: **Answer Key** Date: **4.NBT.5**

Multiplication By 2-Digits

Find the product.

48 x 43 = 2,064	65 x 24 = 1,560

C

I can... multiply two 2-digit numbers.

How well do you understand this skill?

Appendix D

Teacher-Created Independent Practice Quiz for Addition and Subtraction

Name _____ Date _____

Check My Progress (Lessons 5 through 7)

Add.

1. 2,287 + 5,762 — 1. _____

2. 2,632 + 7,814 — 2. _____

3. $42,499 + $38,389 — 3. _____

4. 371,749 + 129,742 — 4. _____

Subtract. Use the back of the paper if you need more room to do your work.

5. $9,975 − $3,368 — 5. _____

6. 3,451 − 756 — 6. _____

7. $77,000 − $31,470 — 7. _____

8. 62,500 − 19,689 — 8. _____

9. 127,019 − 101,200 — 9. _____

10. 660,000 − 83,259 — 10. _____

Solve.

11. There is a mother elephant and a baby elephant at the zoo. The mother elephant weighs 6,934 pounds and the baby weighs 377 pounds. How much do they weigh altogether? (Label!) — 11. _____

12. Miguel wore a pedometer as part of a class experiment. He walked 16,490 steps in 3 days. If he walked 8,245 steps on Monday and 7,516 steps on Tuesday, how many steps did he walk on Wednesday? (Label!) — 12. _____

Appendix D

Teacher-Created Independent Practice Quiz for Multiplication

NAME: _____

LEVEL OF UNDERSTANDING (CIRCLE ONE): 1 2 3 4

Solve each multiplication problem using the method that works best for you!

89 x 3 =	5,677 x 6 =	3,822 x 4 =
83 x 37 =	45 x 19 =	46 x 28 =
56 x 34 =	78 x 41 =	92 x 43 =

Appendix E

Teacher-Created Independent Practice Quiz for Two-by-Two Digit Multiplication

Name: _____

TRADITIONAL METHOD

24 x 35	56 x 72	
47 x 93	68 x 18	85 x 21

AREA MODEL

34 x 68

27 x 92

19 x 53

Appendix F

Teacher-Created Independent Practice Quiz for Multiplication with Varied Problems

NAME: _____

LEVEL OF UNDERSTANDING: 1 2 3 4

WRITE 1- 2 SENTENCES ABOUT WHY YOU ARE AT THIS LEVEL:

_____

_____

15 x 28 =	34 x 52 =	43 x 67 =
81 x 95 =	91 x 68 =	26 x 75 =

Ricky made 22 baskets in the last basketball game. This week, he made twice as many. *About* how many baskets did Ricky make this week?

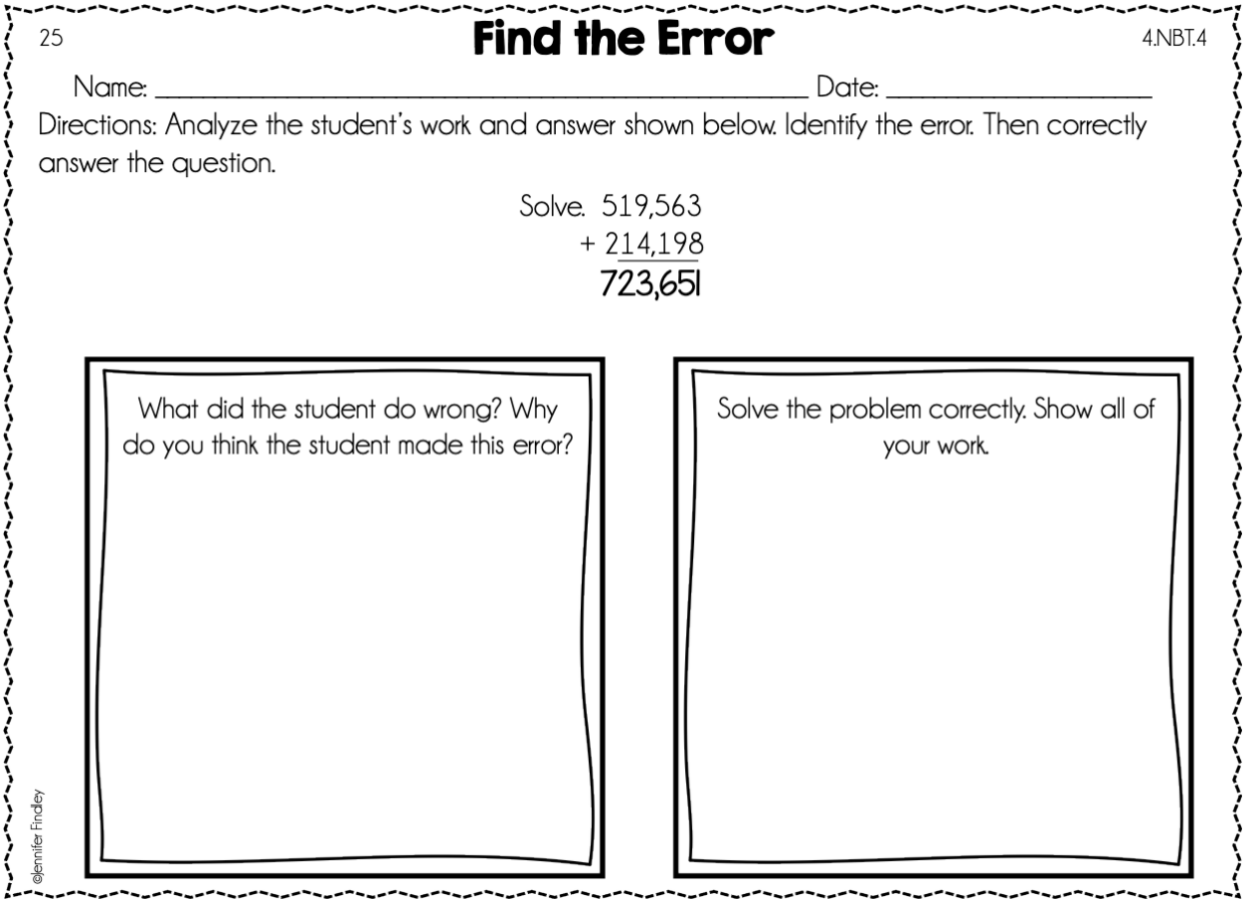
Answer: _____

Craig's goal was to read 50 books during the school year. He reads 15 books before winter break and 21 before spring break. How many books will he need to read after spring break, in order to meet his goal?

Answer: _____

Appendix G

Error Analysis Activities



25

Find the Error

4.NBT.4

Name: _____ Date: _____

Directions: Analyze the student's work and answer shown below. Identify the error. Then correctly answer the question.

Solve. 519,563
+ 214,198
723,651

What did the student do wrong? Why do you think the student made this error?

Solve the problem correctly. Show all of your work.

26

Find the Error

4.NBT.4

Name: _____ Date: _____

Directions: Analyze the student's work and answer shown below. Identify the error. Then correctly answer the question.

Solve.
$$\begin{array}{r} {}^{8}1{}^{16}\,{}^{9}9\,{}^{15}7\,{}^{13}3 \\ -\ 88{,}285 \\ \hline 108{,}778 \end{array}$$

What did the student do wrong? Why do you think the student made this error?

Solve the problem correctly. Show all of your work.

1

Find the Error

4.OA.1

Name: _____ Date: _____

Directions: Analyze the student's work and answer shown below. Identify the error. Then correctly answer the question.

Write *eighteen is six times as many as three* in an equation format.

$18 \times 6 = 3$

What did the student do wrong? Why do you think the student made this error?

Solve the problem correctly. Show all of your work.

27

Find the Error

4.NBT.5

Name: _____ Date: _____

Directions: Analyze the student's work and answer shown below. Identify the error. Then correctly answer the question.

Solve. 1,456
x 8
8,324,048

What did the student do wrong? Why do you think the student made this error?

Solve the problem correctly. Show all of your work.

29

Find the Error

4.NBT.5

Name: _____ Date: _____

Directions: Analyze the student's work and answer shown below. Identify the error. Then correctly answer the question.

Solve.
$$\begin{array}{r} {\scriptstyle 1} \\ 84 \\ \times\ 73 \\ \hline 252 \\ 5780 \\ \hline 6{,}032 \end{array}$$

What did the student do wrong? Why do you think the student made this error?

Solve the problem correctly. Show all of your work.

28

Find the Error

4.NBT.5

Name: _____ Date: _____

Directions: Analyze the student's work and answer shown below. Identify the error. Then correctly answer the question.

Solve. 56×45

$$\begin{array}{r} {}^{2}{}^{3} \\ 56 \\ \times\ 45 \\ \hline 280 \\ 224 \\ \hline 504 \end{array}$$

What did the student do wrong? Why do you think the student made this error?

Solve the problem correctly. Show all of your work.

Appendix H

Institutional Review Board Approval

IRB Protocol 23-092

EIU IRB <eiuirb@eiu.edu>
Wed 9/20/2023 2:23 PM
To:Rachel E Washburn <rewashburn@eiu.edu>
Cc:Sham'ah Md-Yunus <smdyunus@eiu.edu>

September 20, 2023

Rachel Washburn
Sham'ah Md-Yunus
Teaching, Learning, and Foundations

Dear Rachel,

Thank you for submitting the research protocol titled, "Impact of Implementing Error Analysis on Mathematics Proficiency in Fourth Grade" for review by the Eastern Illinois University Institutional Review Board (IRB). The IRB has reviewed this research protocol and effective 9/20/2023, has certified this protocol meets the federal regulations exemption criteria for human subjects research. The protocol has been given the IRB number 23-092. You are approved to proceed with your study.

The classification of this protocol as exempt is valid only for the research activities and subjects described in the above named protocol. IRB policy requires that any proposed changes to this protocol must be reported to, and approved by, the IRB before being implemented. You are also required to inform the IRB immediately of any problems encountered that could adversely affect the health or welfare of the subjects in this study. Please contact the Compliance Coordinator at 581-8576, in the event of an emergency. All correspondence should be sent to the Institutional Research Board, care of the Office of Research and Sponsored Programs.

Thank you for your cooperation, and the best of success with your research.

Institutional Review Board
c/o Office of Research and Sponsored Programs
Telephone: 217-581-8576
Fax: 217-581-7181
Email: eiuirb@eiu.edu

Reviewer Comments: It seems it is an educational approach to use error analysis to help students improve their math skills.
No more than minimal risk to the fourth graders. Good luck with the research!