Mathematics Competition

\$25 prize for the best solution for each of 5 problems.
\$100 prize for solving the most problems throughout the semester.

Problem #4 of five - March 22 to April 5, 2013

These are April Fools problems which are intended to be surprising in some way. Solve as many as you can.

Problem 1. Five students have first names Clark, Donald, Jack, Robin and Steve, and have last names (in a different order) Clarkson, Donaldson, Jackson, Robinson and Stevenson. It is known that Clark is 1 year older than Clarkson, Donald is 2 years older than Donaldson, Jack is 3 years older than Jackson, and Robin is 4 years older than Robinson. Who is older, Steve or Stevenson and what is the difference in their ages? Explain.

Problem 2. A circular clock hanging on a wall has a minute hand which is pointed directly up at all times and a moving hour hand. Nevertheless, the clock shows the correct time at each moment. How could this be and which direction does the hour hand move, clockwise or counterclockwise? Explain.

Problem 3. The sum of five numbers (not necessarily integers) equals 100. The average of the first and the last number is the same as the average of the three numbers in the middle. What is this average? Explain.

Direct any questions to Kamlesh Parwani, OM 3351, or Keith Wolcott, OM 3341

Solved by David Stevens (not a student so does not qualify for prize money).

Solutions. Problem 1: Let C, D, J, R, and S be the ages of Clark, Donald, Jack, Robin and Steve respectively. Let CS, DS, JS, RS, and SS be the ages of Clarkson, Donaldson, Jackson, Robinson and Stevenson respectively. Then we are given that $C = CS+1$, $D = DS+2$, $J = JS+3$, and $R = RS + 4$. We also know that the sum of the five ages is both $C + D + J + R + S$ and $CS+DS+JS+RS+SS$ so these are equal. Taking this equation and subtracting the four previous equations gives that $S = SS - 10$. Thus Stevenson is 10 years older that Steve.

Problem 2: The clock is working correctly, but is rotating counter-clockwise on the wall at exactly the speed which keeps the minute hand pointing upward. Since the clock is rotating at the speed of the minute hand which is faster than the speed of the hour hand, relative to the wall the hour hand is rotating counter-clockwise (relative to the clock, the hour hand is rotating clockwise).

Problem 3: In general, if a proper subset of a finite set of numbers and its complement have the same average, then that average is the same as the average of all of the numbers. In this case the average of all of the numbers is 20.

A short proof of the above general statement follows.

Suppose the numbers are $x_1, x_2, \ldots, x_n$ and the average of the first m and last $n-m$ numbers is A. Then $\sum_{i=1}^{m} x_i = mA$ and $\sum_{i=m+1}^{n} x_i = (n-m)A$. Adding these gives that $\sum_{i=1}^{n} x_i = nA$ so the average of all of the numbers is also A.