Mathematics Competition

$25 prize for the best solution for each of 5 problems.
$100 prize for solving the most problems throughout the semester.

Solved by David Stevens (not a student so does not qualify for prize money).

Solution. Flatten the cube out as shown in the figure. Let the nest be at vertex N and note that when the cube is folded, that the two vertices labeled N are the same vertex. When the spider leaves the nest, it must choose to go onto face A, B, or C. If heading to face E, then initially starting in face C is a poor choice since it is on the opposite side of the cube (N is 1 unit away from face E, and any interior point of face C is more than a unit away from face E, so the spider would be going in the wrong direction). The two circles drawn with radius 2 centered at each N show the range when the spider chooses to start in face A or face B. Thus we have the red area on face E that the spider cannot reach. By the symmetry of the cube, the areas that the spider cannot reach on faces D and F are the same and are also shown in red.
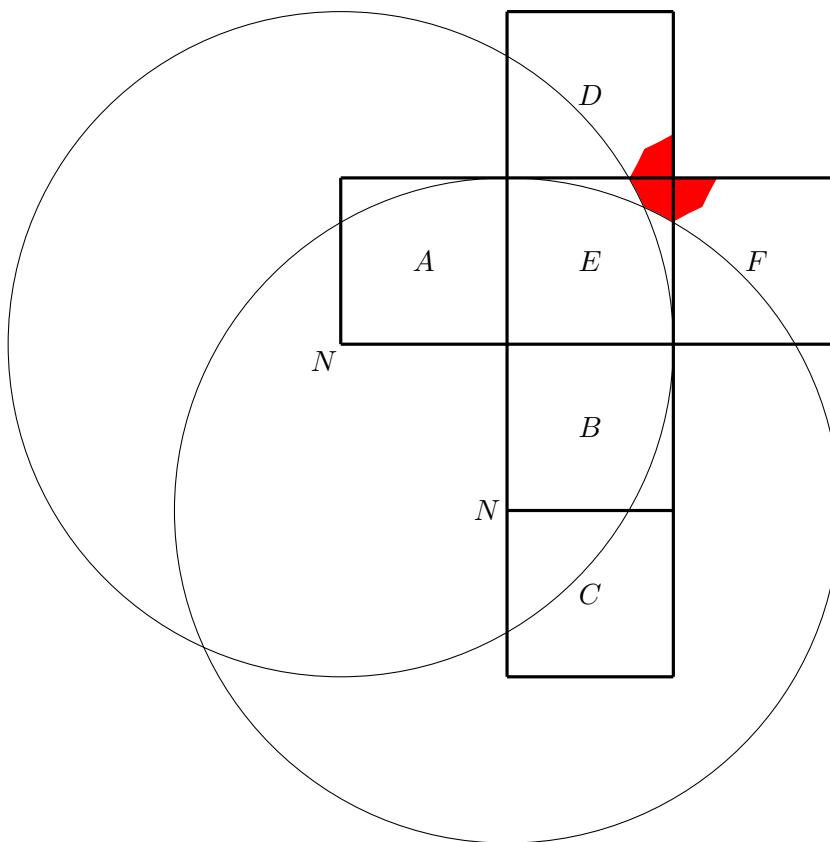

Figure 1: Unit Cube.