Mathematics Competition

$25 prize for the best solution for each of 5 problems.
$100 prize for solving the most problems throughout the semester.

Problem #2 of five - February 8 to Feb 22, 2012

Answer both of the following questions.

1) Two boats on the opposite shores of a river start moving towards each other. When they pass each other they are 750 yards from one shoreline. They each continue to the opposite shore, immediately turn around and start back. When they meet again they are 250 yards from the other shoreline. Each boat maintains a constant speed throughout. How wide is the river?

2) Two people stand back to back next to the rails in a small railway station. As the head of the express train that passes the station reaches them, they start to walk parallel to the rails. As the tail of the train reaches each of them, they stop, having walked 30m and 40m respectively. If they both walked with identical, constant speed and the train kept its speed as well, can you tell how long the train was?

Direct any questions to Kamlesh Parwani, OM 3351, or Keith Wolcott, OM 3341

Correct solutions were submitted by Jason Haarmann and David Stevens. The $25 prize has been awarded to Jason Haarmann.

Solution. 1) Let w be the width of the river, and let a and b be the distances covered by the boats by the time they first pass each other.

The sum of these distances is the width of the river, so

$$w = a + b \tag{1}$$

We're told that the boats first pass 750 yards from one shoreline, which means one of the boats traveled 750 yards. Let's say without loss of generality that

$$a = 750 \tag{2}$$

By the second passing, each boat has covered the width of the river, and turned around. Then together, the boats have covered the width of the river once more, so the sum of the distances they've traveled is three times with width of the river. Since they travel at a constant rate, and together they've gone three times as far as when they first passed, it follows that one of them has traveled a distance of $3a$ and the other has traveled $3b$.

When the boats passed a second time, 250 yards from the "other" shoreline, it follows that the same boat that had traveled 750 yards by their first passing has traveled $w + 250$ yards by the second passing.

$$3a = w + 250 \tag{3}$$

Now we have three equations in three unknowns, so it is a straightforward task to obtain solutions.

$$w = 2000 \quad a = 750 \quad b = 1250$$

So the length of the river is 2000 yards.

2) Let A be the person moving in the same direction as the train. Note that person A traveled 10m while the tail of the train passed from one person to the other, a distance of 70m. So the train is traveling at seven times the speed of A. Now A walks 40m, and so, the train travels $40 \times 7 = 280$m in the same time. However, during this time interval, person A moves 40m in the direction of the train. This means that the length of the train is $280 - 40 = 240$m.