Mathematics Competition

Problem #2 of five - September 27 to October 10, 2013

a) An ant is crawling on the plane horizontally or vertically (changing direction from horizontal to vertical and vice versa). The ant starts and ends its path at the same point A. As a result, the ant's path is a closed broken line with n segments.
Can n be equal to one of the following numbers: 2012, 2013, 2014, 2015, or 2016? If yes, which one(s)? If yes, show or explain the ant's path with this number of n segments; if not, explain why such a path cannot exist.

b) A bug in three dimensional space can fly parallel to the x, the y, or the z-axis (changing direction from parallel to one axis to parallel to another axis). It starts and ends its path at the same point A. As a result, the bug's path is a closed spatial broken line with n segments.
Can n be equal to one of the following numbers: 2012, 2013, 2014, 2015, or 2016? If yes, which one(s)? If yes, show or explain the bug's path with this number of n segments; if not, explain why such a path cannot exist.

Direct any questions to Gregory Galperin, OM 3361, or Keith Wolcott, OM 3341

Congratulations to our winner Jerry Bragg!

Solution. When we stated the problems, we intended that the paths would not have any self-intersections, but we didn't say this formally. Thus we give solutions for both with and without this assumption.

a) First suppose that the path has no self-intersections. The closed broken path with n segments alternates horizontal and vertical so there must be the same number of horizontals as verticals and thus n is even. Figure 1 illustrates any even number $n \geq 4$. Therefore 2012, 2014, and 2016 are possible and are the only possibilities in the given list.

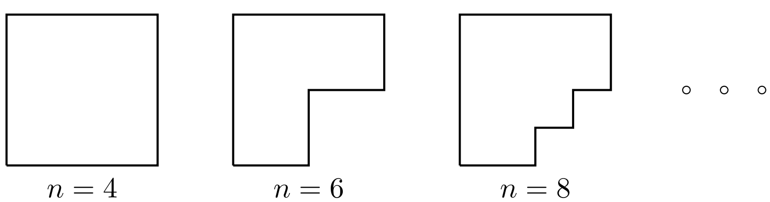

Figure 1: Even $n \geq 4$.

a′) Now suppose that self-intersections are allowed. As above, an even number $n \geq 4$ is possible and Figure 2 illustrates odd $n \geq 5$. The path starts at A, with the first segment from A

to C and the last segment from B to A, and are thus counted. Therefore, all of the given numbers 2012, 2013, 2014, 2015, and 2016 are possible in this case.

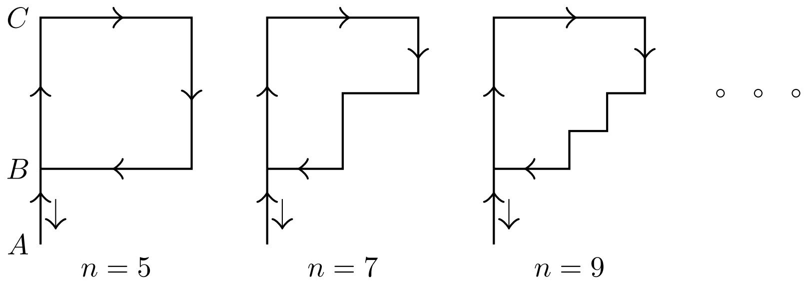

Figure 2: Odd $n \geq 5$.

b) Again, first assume that the path has no self-intersections. By just using the $xy-$plane, n can be any even integer larger than 2, as it was shown in Figure 1. Figure 3 shows how to add three segments (the three in the $yz-$plane) to each of the paths in Figure 1 which means that n can also be any odd $n \geq 7$. Therefore, all of the given numbers 2012, 2013, 2014, 2015, and 2016 are possible in this case.

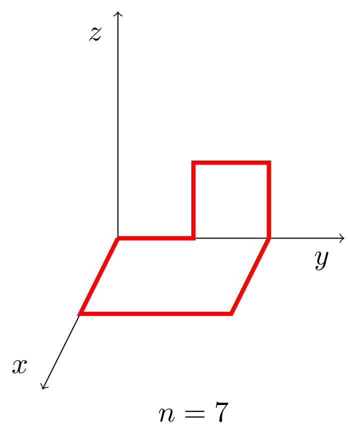

Figure 3: Odd $n \geq 7$.

b′) With the assumption that **there may be self-intersections**, all of the given numbers 2012, 2013, 2014, 2015, and 2016 are also possible since it can be done the same as in case b) above without self-intersections.