Mathematics Competition

April's fools Problem #4 - March 31 to April 11, 2014

1. You stand next to a mirror pillar at Macy's. The cross-section of the pillar is a square. Is it possible to stand in such a position that you cannot see yourself in the pillar's four mirrors (i.e. you are completely invisible for yourself) while at the same time you can see the background? Justify your answer.

2. The quadratic equation $x^2 + ax + b = 0$ has roots a and b. Find a and b.

   Justify your answers, that is prove that you found all possible pairs (a, b) satisfying the formulation.

3. A ray of light enters a very small hole of a container. The light starts reflecting on the container's interior mirror surface by the law "*the angle of incidence equals the angle of reflection*". Can the ray's trajectory become periodic eventually? Justify your answer.

4. A barrel of cucumbers weighs 100 kg. Each cucumber consists of 99% of water. After a while, some water was evaporated so that after the evaporation each cucumber consists of 98% of water. What is the new weight of the cucumbers in the barrel? Justify your answer.

5. Baron Munchausen is a very famous and popular character of many stories in Germany. Usually he doesn't tell the whole truth in his stories, but sometimes he does tell the truth. Once baron Munchausen made the following 7 statements about the numbers

$$A = \Big(1\frac{1}{10}\Big) \cdot \Big(1\frac{1}{11}\Big)\Big(\cdot 1\frac{1}{12}\Big) \cdot \ \ \ldots \ \ \cdot \Big(1\frac{1}{98}\Big) \cdot \Big(1\frac{1}{99}\Big)$$

   and

$$B = \Big(1\frac{1}{100}\Big) \cdot \Big(1\frac{1}{101}\Big)\Big(\cdot 1\frac{1}{102}\Big) \cdot \ \ \ldots \ \ \cdot \Big(1\frac{1}{998}\Big) \cdot \Big(1\frac{1}{999}\Big) :$$

   - A is an integer;
   - B is an integer;
   - $A + B$ is an integer;
   - $A - B$ is an integer;
   - A/B is an integer;
   - B/A is an integer;
   - $A = B$.

   How many times did Baron Munchausen tell the truth? Justify your answer.

6. n flies in a cubic room sit at the 6 room's surfaces: the 4 walls, the floor, and the ceiling. It turned out that the number of flies on each surface is the same, $k \geq 1$. Remember that no two flies are at the same point.

   (a) What the smallest number of flies, n, when $k = 1$?

   (b) What the smallest number of flies, n, when if $k = 2$?

   Justify your answer.

Direct any questions to Gregory Galperin, OM 3361.

Rules and Awards

- Any undergraduate currently enrolled at EIU is eligible to participate.
- Each solution is to be the work of one individual and is to be submitted with the solver's name, year in school, email address, local address and home address.
- Each solution is to be written or typed and is due in the main Mathematics Department office (OM3611) by 2:00 p.m., Friday, February 28, 2014.
- Entries will be judged on the basis of clarity of exposition and elegance of solution.
- Up to $40 will be distributed as prize money for each problem. It will be distributed based on the quality of the solutions, but roughly, an award of $10 will be awarded for the best solutions and $5 will be awarded for partial solutions. In the case that there are many correct solutions, we will have a drawing for the prizes and in case no award is made, the prize money will be added to the next problem's prize fund.
- **Challenges, solutions, names of all solvers, and comments will be posted on the Challenge of the Week homepage:**

  `http://www.eiu.edu/math/challenge.php`.