

# Mathematics Competition

**\$25 prize** for the best solution for each of 5 problems.

**\$100 prize** for solving the most problems throughout the semester.

## *Problem #1 of five - Aug 31 to Sept 14, 2012*

A pet shop owner has \$100 to buy 100 animals. Geckos cost \$5 each, rabbits cost \$3, and goldfish cost 5 cents each. How many of each does the owner have to buy if she must buy at least one of each and pay exactly \$100 for the 100 pets?

Find all solutions and show that you have found all solutions.

*Direct any questions to Kamlesh Parwani, OM 3351, or Keith Wolcott, OM 3341*

**Correct solutions were submitted by Kyle Davis, Brian Wood, David Stevens, Wen Li Yao, Joel Blome, and Kristi Goldstein.** The \$25 prize has been split between Kyle Davis and Brian Wood. Congratulations to all of the solvers!

**Solution.** If  $G$ ,  $R$ , and  $F$  are the number of geckos, rabbits, and fish that are bought, we have the following two equations.

$$G + R + F = 100$$

$$5G + 3R + .05F = 100$$

Solving the first equation for  $G$  and substituting into the second equation gives

$$5(100 - R - F) + 3R + .05F = 100.$$

Multiplying by 20 in order to get all integers gives

$$100(100 - R - F) + 60R + F = 2000.$$

Simplifying gives

$$40R + 99F = 8000.$$

In this equation,  $40R$  and 8,000 are divisible by 40, so  $99F$  must be divisible by 40. Since 99 and 40 have no common factors  $F$  must be divisible by 40. The only multiples of 40 that are strictly between 0 and 100 are 40 and 80. If  $F = 40$ , solving for  $R$  gives  $R = 101$  which is too large. Thus the only possibility is for  $F = 80$ . In this case,

$$R = \frac{8000 - 99(80)}{40} = 200 - 99(2) = 2$$

and

$$G = 100 - R - F = 100 - 2 - 80 = 18.$$

Therefore, the petshop owner must buy 18 geckos, 2 rabbits, and 80 goldfish.