PROBLEM # 2

It is possible to place the integers 1 through 32 inclusive around a circle so that each number has two neighbors and every neighboring pair sums to a perfect square. For example, one may place them in the following order:

1 15 10 26 23 2 14 22 27 9 16 20 29 7 18 31 5 11 25 24 12 13 3 6 30 19 17 32 4 21 28 8 1

where the first and last 1 are the same.

(a) Is it possible to place the integers 1 through 33 inclusive around a circle so that each number has two neighbors and every neighboring pair sums to a perfect square? Give an example of this, or explain why none can exist.

(b) Is it possible to place the integers 1 through 30 inclusive around a circle so that each number has two neighbors and every neighboring pair sums to a perfect square? Give an example of this, or explain why none can exist.

Direct any questions to
Grant Lakeland (OM 3226)

Rules & Rewards

• Any undergraduate currently enrolled at EIU is eligible to participate.

• Each solution is to be the work of one individual and is to be submitted with the solver’s name, year in school, email address, local address, and home address.

• Each solution is to be written or typed and is due in the main Mathematics Department office (OM 3611) by 2:00pm, Friday, April 7, 2023.

• Entries will be judged on the basis of clarity of exposition and elegance of the solution. That is to say, the explanation is more important than the answer.

• An award of $50 will be given for the best solution. In the case of a two-way tie, the award will be evenly split. If there are more than two ‘best’ solutions, a drawing will be held for the reward. In the case no award is made for this week’s challenge, $50 will be added to the next week’s award.

• Names of all solvers will be posted on the Challenge of the Month bulletin board and on the Challenge homepage: http://www.eiu.edu/math/challenge.php