PROBLEM # 1

1000 small boxes located in one row are numbered from left to right from 1 to 1000. At each of the following steps, we put candies in some of the boxes or take them out of some other boxes.

At the first step, we put one candy in each box. At the second step, we take out the candies from all the boxes whose numbers are even: # 2, 4, 6, ... , 1000. At the third step, we have deal with all the boxes whose numbers are divisible by 3: # 3, 6, 9, ..., 999, and if there is no candy in a box, then we put a candy into that box, and if a box contains a candy, we take this candy out of the box. At the fourth step, we consider all the boxes whose numbers are divisible by 4: # 4, 8, 12, ..., 996, 1000, and we put a candy in the box if this box is empty, and if a candy is there, we remove this candy from the box. At the fifth step, as before, we put a candy in or take it out from each box whose number is divisible by 5: # 5, 10, 15, ..., 995, 1000. We repeat the same procedure further, on the 6th, 7th, etc. step until the very last, the 1000th step (where we have deal with only one box, # 1000).

How many boxes will contain candies after making all the 1000 steps? What are the numbers of these boxes? Explain your answer.

Direct any questions to
Grant Lakeland (OM 3226)

Rules & Rewards

• Any undergraduate currently enrolled at EIU is eligible to participate.

• Each solution is to be the work of one individual and is to be submitted with the solver’s name, year in school, email address, local address, and home address.

• Each solution is to be written or typed and is due in the main Mathematics Department office (OM 3611) by 2:00pm, Monday, February 20, 2023.

• Entries will be judged on the basis of clarity of exposition and elegance of the solution. That is to say, the explanation is more important than the answer.

• An award of $50 will be given for the best solution. In the case of a two-way tie, the award will be evenly split. If there are more than two ‘best’ solutions, a drawing will be held for the reward. In the case no award is made for this week’s challenge, $50 will be added to the next week’s award.

• Names of all solvers will be posted on the Challenge of the Month bulletin board and on the Challenge homepage: http://www.eiu.edu/math/challenge.php