University Math Challenge

PROBLEM # 1

There is an empty box. Put in the empty box, side by side with each other, 4 empty boxes. (Now the first box is no longer empty.) At the next step, and each step thereafter, for each empty box, **either** place 4 empty boxes side by side inside it, making it no longer empty, **or** leave the box empty.

Question 1. When there are 10 non-empty boxes, how many empty boxes are there?

Question 2. When there are 100 empty boxes, how many non-empty boxes are there?

Now change the number of empty boxes placed inside each box from 4 to 5.

Question 3. When there are 50 non-empty boxes, how many empty boxes are there?

Question 4. Is it possible that there could be 100 empty boxes? If so, how many non-empty boxes are there at that point? If not, why not?

Explain your answers.

Rules & Rewards

- Any undergraduate currently enrolled at EIU is eligible to participate.

- Each solution is to be the work of one individual and is to be submitted with the solver's name, year in school, email address, local address, and home address.

- Each solution is to be written or typed and is due in the main Mathematics Department office (OM 3611) by 2:00pm, Friday, February, 4, 2022.

- Entries will be judged on the basis of clarity of exposition and elegance of the solution. That is to say, the *explanation* is more important than the answer.

- An award of $50 will be given for the best solution. In the case of a two-way tie, the award will be evenly split. If there are more than two 'best' solutions, a drawing will be held for the reward. In the case no award is made for this week's challenge, $50 will be added to the next week's award.

- Names of all solvers will be posted on the Challenge of the Month bulletin board and on the Challenge homepage: `http://www.eiu.edu/math/challenge.php`