

# University Math Challenge

November 9, 2023 to December 8, 2023

## PROBLEM # 2

Consider a  $4 \times 4$  square grid. It has 16 squares, or “cells”. Each cell has several adjacent cells which we call neighbors or neighboring cells (neighboring cells have a common side, not just a vertex, i.e. they are vertically or horizontally adjacent, not diagonally). Each corner cell has 2 neighbors, each cell on the edge but not in the corner has 3 neighbors, and each cell in the central  $2 \times 2$  square has 4 neighbors. A number is written in each of the 16 cells in such a way that for any cell, the sum of the numbers in its neighboring cells is the same. (Thus there are 16 “neighboring” sums, one sum per each cell, and all 16 sums are equal to each other.)

(A). Suppose all the 16 “neighboring” sums are equal to 1. What is the sum  $\Sigma$  of all the 16 numbers in the table? Find all possible answers.

(B). Suppose now that all the 16 table numbers are **distinct positive integers** and denote each of the 16 equal “neighboring” sums as  $N$ . In this case, what could be the smallest/least value for the “neighboring” sum  $N$ ? And what could be the smallest/least sum of all the 16 distinct integers?

Justify your answers!

*Direct any questions to  
Grant Lakeland (OM 3226)*

## Rules & Rewards

- Any undergraduate currently enrolled at EIU is eligible to participate.
- Each solution is to be the work of one individual and is to be submitted with the solver’s name, year in school, email address, local address, and home address.
- Each solution is to be written or typed and is due in the main Mathematics Department office (OM 3611) by 2:00pm, Friday, December 8, 2023.
- Entries will be judged on the basis of clarity of exposition and elegance of the solution. That is to say, the *explanation* is more important than the answer.
- An award of \$50 will be given for the best solution. In the case of a two-way tie, the award will be evenly split. If there are more than two ‘best’ solutions, a drawing will be held for the reward. In the case no award is made for this week’s challenge, \$50 will be added to the next week’s award.
- Names of all solvers will be posted on the Challenge of the Month bulletin board and on the Challenge homepage: <http://www.eiu.edu/math/challenge.php>