PROBLEM # 1

Some number \(n > 10\) boxes numbered, in order, from \#1 to \#n, are arranged around a circle. Each box contains some number of small balls inside it. Among them, it is known that there is a box with 8 balls, and that there is another box with either 17 or 23 balls, though which of these is unknown. You make the following \(n\) transfers from box to box. First you move 1 ball from box \#1 to box \#2; then move 2 balls from box \#2 to box \#3; then move 3 balls from box \#3 to box \#4; and so on. The last two transfers are the following: you move \((n - 1)\) balls from box \(#(n - 1)\) to box \#n, and, finally, you move \(n\) balls from box \#n to box \#1.

It is known that after making all the \(n\) transfers, the number of balls in each box is the same.

Answer the following questions, with justification:

(a) Which box contained 8 balls in the beginning?
(b) How many balls were in the other box at the beginning, 17 or 23?
(c) How many boxes were there? In other words, find \(n\) (all possible values).
(d) How many balls were in each box in the very end (after making all the \(n\) transfers)?

Direct any questions to
Grant Lakeland (OM 3226)

Rules & Rewards

• Any undergraduate currently enrolled at EIU is eligible to participate.

• Each solution is to be the work of one individual and is to be submitted with the solver’s name, year in school, email address, local address, and home address.

• Each solution is to be written or typed and is due in the main Mathematics Department office (OM 3611) by 2:00pm, Friday, Octobr 13, 2023.

• Entries will be judged on the basis of clarity of exposition and elegance of the solution. That is to say, the explanation is more important than the answer.

• An award of $50 will be given for the best solution. In the case of a two-way tie, the award will be evenly split. If there are more than two ‘best’ solutions, a drawing will be held for the reward. In the case no award is made for this week’s challenge, $50 will be added to the next week’s award.

• Names of all solvers will be posted on the Challenge of the Month bulletin board and on the Challenge homepage: http://www.eiu.edu/math/challenge.php