

University Math Challenge

February 3, 2020 to February 28, 2020

PROBLEM 1

Let P be a 16-digit **PRIME** number (so P is not divisible by any integer strictly between 1 and P). If we insert an arbitrary non-zero digit x between P 's digits, or attach the digit x in front of or at the end of P , we obtain seventeen 17-digit integers Q_1, Q_2, \dots, Q_{17} . Let's call x a **curious digit** if the seventeen integers obtained are all **COMPOSITE**. (An integer Q is composite if it's divisible by some integer strictly between 1 and Q .)

- (a) Prove that for every 16-digit prime number P , a curious digit x exists.
- (b) Is there necessarily more than one curious digit for each such prime number P ? If yes, prove it. Otherwise, find a prime P for which exactly one curious digit exists.

*Direct any questions to
Grant Lakeland (OM 3226) or Gregory Galperin (OM 3361)*

Rules & Rewards

- Any undergraduate currently enrolled at EIU is eligible to participate.
- Each solution is to be the work of one individual and is to be submitted with the solver's name, year in school, email address, local address, and home address.
- Each solution is to be written or typed and is due in the main Mathematics Department office (OM 3611) by 2:00pm, Friday, February 28, 2020.
- Entries will be judged on the basis of clarity of exposition and elegance of the solution. That is to say, the *explanation* is more important than the answer.
- An award of \$50 will be given for the best solution. In the case of a two-way tie, the award will be evenly split. If there are more than two 'best' solutions, a drawing will be held for the reward. In the case no award is made for this week's challenge, \$50 will be added to the next week's award.
- Names of all solvers will be posted on the Challenge of the Month bulletin board and on the Challenge homepage: <http://www.eiu.edu/math/challenge.php>