Problem of the Week


John subtracted from a regular irreducible fraction with the numerator 17 the closest, but smaller, irreducible fraction with numerator 1. The difference that John obtained was a positive irreducible fraction \( \frac{p}{q} \). Can \( p \), the numerator of this fraction,

(i) exceed 17?
(ii) be equal to 17?

Notes:
- A positive fraction \( \frac{a}{b} \) is regular if \( a < b \). For example, \( \frac{10}{13} \) and \( \frac{17}{235} \) are regular but \( \frac{13}{10} \) and \( \frac{333}{17} \) are not.
- A fraction \( \frac{a}{b} \) is irreducible if \( \gcd(a, b) = 1 \). For example, \( \frac{10}{13} \) and \( \frac{17}{235} \) are irreducible but \( \frac{10}{17} \) and \( \frac{17}{235} \) are not as they can be reduced to \( \frac{2}{7} \) and \( \frac{1}{15} \), respectively.

Direct any questions to
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Rules & Rewards

- Any undergraduate currently enrolled at EIU is eligible to participate.
- Each solution is to be the work of one individual and is to be submitted with the solver’s name, year in school, email address, local address, and home address.
- Each solution is to be written or typed and is due in the main Mathematics Department office (OM 3611) by 2:00pm, Friday, 24. October 2014.
- Entries will be judged on the basis of clarity of exposition and elegance of the solution. That is to say, the explanation is more important than the answer.
- An award of $20 will be given for the best solution. In the case of a two-way tie, the award will be evenly split. If there are more than two ‘best’ solutions, a drawing will be held for the aware. In the case no award is made for this week’s challenge, $20 will be added to the next week’s award.
- Names of all solvers will be posted on the Challenge of the Week bulletin board and on the Challenge of the week homepage: http://www.eiu.edu/math/challenge.php