

Problem of the Week

Problem #3: 10. October 2014 to 24. October 2014

John subtracted from a regular irreducible fraction with the numerator 17 the closest, but smaller, irreducible fraction with numerator 1. The difference that John obtained was a positive irreducible fraction $\frac{p}{q}$. Can p , the numerator of this fraction,

- (i) exceed 17?
- (ii) be equal to 17?

Notes:

- A positive fraction $\frac{a}{b}$ is *regular* if $a < b$. For example, $\frac{10}{13}$ and $\frac{17}{235}$ are regular but $\frac{13}{10}$ and $\frac{235}{17}$ are not.
- A fraction $\frac{a}{b}$ is *irreducible* if $\gcd(a, b) = 1$. For example, $\frac{10}{13}$ and $\frac{17}{235}$ are irreducible but $\frac{10}{14}$ and $\frac{17}{255}$ are not as they can be reduced to $\frac{5}{7}$ and $\frac{1}{15}$, respectively.

*Direct any questions to
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Rules & Rewards

- Any undergraduate currently enrolled at EIU is eligible to participate.
- Each solution is to be the work of one individual and is to be submitted with the solver's name, year in school, email address, local address, and home address.
- Each solution is to be written or typed and is due in the main Mathematics Department office (OM 3611) by 2:00pm, Friday, 24. October 2014.
- Entries will be judged on the basis of clarity of exposition and elegance of the solution. That is to say, the *explanation* is more important than the answer.
- An award of \$20 will be given for the best solution. In the case of a two-way tie, the award will be evenly split. If there are more than two 'best' solutions, a drawing will be held for the aware. In the case no award is made for this week's challenge, \$20 will be added to the next week's award.
- Names of all solvers will be posted on the Challenge of the Week bulletin board and on the Challenge of the week homepage: <http://www.eiu.edu/math/challenge.php>