Department of Mathematics and Computer Science

Friday, February 24, 2017, 4:10 pm

COLLOQUIUM TALK

Speaker: Gregory Galperin (EIU)

Old Main 2231

The "Look and Say Sequence", Conway's constant, the 92 Audio-active elements, and the Cosmological Theorem

Abstract: One of the most intriguing sequences, the so-called "*Look and Say Sequence*" or the *C-sequence*, is Conway's

$$1,\ 11,\ 21,\ 1211,\ 111221,\ 312211,\ 13112221,\ \dots\ .$$

It is created, term by term, by the *audioactive operator* $\mathcal{A}$ which transforms each term to the next one by "reading" the previous term as follows:

$$1 \implies \textit{one } 1 \implies \textit{two } 1's \implies \textit{one } 2 \textit{ one } 1 \implies \textit{one } 1 \textit{ one } 2 \textit{ two } 1's \implies \dots\ .$$

The C-sequence has many interesting properties, some of which I will touch in my talk.

Q: *Can a C-number has 2017 digits? When* 4 *will show up in some C-number?*

The remarkable thing about the C-sequence is that even though it seems at first glance to be quite arbitrary and non-mathematical, it has some interesting properties unearthed by John Conway. Most notably, he showed that the number of digits in each term on average grows by about 30% from one term to the next; the exact result is formulated as the following

Cosmological Theorem. *Let d_n stand for the length of the nth C-number. Then*

(a) *the limit* $\lambda := \lim_{n\to\infty} d_{n+1}/d_n$ *exists;*

(b) *the number λ is the unique positive real root of a very specific polynomial of degree 71.*

The number $\lambda \approx 1.303577269\dots$ is known as *Conway's constant*.

The crucial point in Conway's proof is that each C-number is made up of one or more of 92 "basic" non-interacting strings (subsequences), or audioactive "elements", called him as the basic 92 chemical elements, from Hydrogen H to Uranium U.

From mathematical point of view, the C-numbers are vectors in 92-dimensional space, the operator $\mathcal{A}$: $\mathbb{R}^{92} \to \mathbb{R}^{92}$ has a 71-dimensional subspace on which it is irreducible, and the Conway constant λ is actually the maximal positive eigenvalue of the operator A.

SNACKS IN FACULTY LOUNGE AT 3:30 PM.
EVERYONE WELCOME (EVEN IF YOU ARE UNABLE TO ATTEND THE TALK)