Provide induction proofs for each of the following, using the same technique presented in lecture. Be sure to label all parts and include reasons for each step.

Friday (10/5)	Monday (10/8)	Wednesday (10/10)	Friday (10/12)	Monday (10/15)
Section 3.1	1–5	6–10	11–15	16–20

1. $11^n - 4^n$ is divisible by 7 $\forall\ n \geq 0$
2. $\sum_{i=1}^{n}(2i-1) \ = \ n^2 \ \ \forall\ n \geq 1$
3. Find the sum (in closed form) and prove your claim using induction. Hint: rewrite as a summation. $1(2) + 2(3) + 3(4) + \ldots + n(n+1) \ = \ ?$
4. $\sum_{i=1}^{n} 5 \ = \ 5n \ \ \forall\ n \geq 1$
5. $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} < 2 - \frac{1}{n} \ \ \forall\ n \geq 2$
6. Prove, using induction, that given $n > 0$ boxes and $n+1$ balls, when the balls are put into the boxes (in any configuration), at least one box has more than one ball in it. This is known as the Pigeonhole Principle.
7. $n^3 - n$ is divisible by 6 $\forall\ n \geq 0$
8. $n^2 - 7n + 12 \geq 0 \ \ \forall\ n > 3$
9. $\sum_{i=1}^{n} i2^i \ = \ (n-1)2^{n+1} + 2 \ \ \forall\ n \geq 1$
10. $\sum_{i=0}^{n}(2i+1)^2 \ = \ \frac{(n+1)(2n+1)(2n+3)}{3} \ \ \forall\ n \geq 0$
11. $\sum_{i=0}^{n} 3(5^i) \ = \ \frac{3(5^{n+1}-1)}{4} \ \ \forall\ n \geq 0$
12. $\sum_{i=1}^{n} \frac{1}{n} \ = \ \frac{2}{n(n+1)} \ \ \forall\ n \geq 1$
13. $\sum_{i=1}^{n} i2^{i-1} \ = \ (n-1)2^n + 1 \ \ \forall\ n \geq 1$
14. $n^5 - n$ is divisible by 5 $\forall\ n \geq 0$
15. $n^2 - 1$ is divisible by 8 $\forall$ *odd* $n > 0$
16. $\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} \ = \ \frac{n}{2n+1} \ \ \forall\ n \geq 1$
17. For which non–negative integers n is $2n + 3 \leq 2^n$? Prove your answer correct using mathematical induction.

18. Prove by induction that $T(n) = 3n + 2$ if

$$T(n) = \begin{cases} 2 & n = 0 \\ 3 + T(n-1) & n > 0 \end{cases}$$

19. Guess the closed form of the following and prove your guess correct by induction. It will be most helpful to solve $T(n)$ for $n = 1$, 2, 3, 4, and 5 in order to see a pattern.

$$T(n) = \begin{cases} 1 & n = 1 \\ n + \sum_{i=1}^{n-1} T(i) & n > 1 \end{cases}$$

20. Uset the Master Method given in class to solve the following recurrence relations, if possible:

    (a) $T(n) = 7T(\frac{n}{2}) + 18n^2$

    (b) $T(n) = 3T(\frac{n}{4}) + 5n \log n$

    (c) $T(n) = 9T(\frac{n}{3}) + 12n^5$

    (d) $T(n) = 2T(\frac{n}{4}) + 2^{n+1}$