# Department of Mathematics and Computer Science 

Friday, October 20, 2017, 4:10 pm<br>COLLOQUIUM TALK<br>Speaker: Peter Andrews (EIU)<br>Old Main 2231

## What If Your Directrix Were a Circle?


#### Abstract

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In his recent talk, Charles Delman went from the classical (and I mean really classical) definition of a conic section as a planar slice through a cone to an equivalent definition in terms of a focus (a single point) and a directrix (a line not going through that point. As you may recall, each conic section is uniquely defined by its focus, its directrix, and a single non-negative real number - its eccentricity. The purpose of this talk is to explore what kinds of curves one might get if you took a similar definition but used a circle as a directrix rather than a straight line. The results are simple, elegant, and rewarding in the sense that they provide lovely proofs of some fundamental properties of these loci.


The material requires no extensive background in analytic geometry and should be accessible to undergraduates as well as graduate students and faculty.

Homework: Solutions will not be collected or graded, but you may called upon to describe your solution(s).

1. Recall that a parabola is a conic section with eccentricity 1 , meaning it is the locus of points which are equidistant from the focus and the directrix (the latter distance measured perpendicularly to the line). Suppose you have two parabolas which share the same directrix $\ell$ but have different foci - both on the same side of $\ell$. Generically these two parabolas intersect in two points. What is the locus of these points as you move the directrix, keeping it parallel to its original position?
2. Suppose you have a fixed circle with center $O$, a fixed point $Y$ outside the circle, and a movable point $X$ constrained to lie on the circle. Is there a position for $X$ so that the perpendicular bisector of the segment $X Y$ is parallel to the radial ray $O X$ ?
