Department of Mathematics and Computer Science

Friday, October 30, 2015, 4:10 pm

COLLOQUIUM TALK

Speaker: A.J. Hildebrand, UIUC
Old Main 2231

How to Sum $1 - 1 + 1 - 1 + \cdots$: A Journey through the World of Divergent Series from Euler to the Present

Abstract:

What is $1-1+1-1+1-1+\cdots$? How about $1+2+4+8+16+\cdots$, or $1+2+3+4+5+\cdots$, or $1!-2!+3!-4!+5!-6!+\cdots$? Of course, we know from calculus that these series are divergent, so asking for their sum does not seem to make sense. Yet, before the advent of calculus and modern notions of convergence, people freely worked with divergent series and attempted to evaluate them. In particular, Euler obtained the values 1/2, -1, -1/12, and 0.596347362123, respectively, for the four series above. How did Euler come up with these values?

In this talk, we will take a journey through a fascinating underworld—the theory of "Divergent Series"—that provides a rigorous mathematical foundation for Euler's reasoning. Along the way, we will encounter surprising results, spooky phenomena, as well as mysteries that have yet to be fully explained and have been the subject of a recent undergraduate research project at the Illinois Geometry Lab.