Department of Mathematics and Computer Science

Friday, March 6, 2015, 4:10 pm

COLLOQUIUM TALK

Speaker: Bogdan Petrenko

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Coefficients of cyclotomic polynomials.

Abstract:

The first 4 cyclotomic polynomials are:

$$\Phi_1(x) = x - 1,\ \Phi_2(x) = x + 1,\ \Phi_3(x) = x^2 + x + 1,\ \Phi_4(x) = x^2 + 1.$$

Therefore, we can write

$$x - 1 = \Phi_1(x),\ x^2 - 1 = \Phi_1(x)\,\Phi_2(x),\ x^3 - 1 = \Phi_1(x)\,\Phi_3(x),\ x^4 - 1 = \Phi_1(x)\,\Phi_2(x)\,\Phi_4(x).$$

In general, cyclotomic polynomials are exactly the irreducible factors over the integers of some polynomial $x^n - 1$ for some positive integer n, and therefore these polynomials frequently appear in algebra and number theory. The nth cyclotomic polynomial $\Phi_n(x)$ for $n \geq 2$ can be defined by the recursive formula

$$\Phi_n(x) = \frac{x^n - 1}{\prod\limits_{d|n,\, d<n} \Phi_d(x)}$$

(here the product is taken over all the positive divisors of n that are less than n). The degree of $\Phi_n(x)$ is $\varphi(n)$ where φ is Euler's totient function. The coefficients of each of the polynomials $\Phi_1(x), \Phi_2(4), \ldots, \Phi_{104}(x)$ can only be 0 or ± 1. Such polynomials are called `flat`. There are infinitely many flat cyclotomic polynomials. In particular, any polynomial $\Phi_m(x)$, where m is the product of at most 2 prime numbers, is flat. The smallest n for which $\Phi_n(x)$ isn't flat is 105 because -2 is the coefficient at both x^7 and x^{41} in $\Phi_{105}(x)$.

Let S denote the set of coefficients of all cyclotomic polynomials. In 1931, Issai Schur proved that S contains all negative even numbers. By carefully analyzing Schur's argument, Jiro Suzuki in 1987 proved that S contains any integer. The goal of this talk is to explain the proof Suzuki's theorem and discuss some related questions.

SNACKS IN FACULTY LOUNGE AT 3:30 PM.
EVERYONE WELCOME (EVEN IF YOU ARE UNABLE TO ATTEND THE TALK)
