

Geometry, Topology, and Dynamics Day 2015

All talks will take place on the second floor in Old Main.

Time	Speaker	Title
9:30 – 10:20	Alexander Blokh	Combinatorial models for spaces of cubic polynomials
10:45 – 11:35	Robert Foote	Inversion in Spherical and Hyperbolic Geometries
12:00 – 2:00	Lunch	
2:00 – 2:50	Florin Boca	Spacing statistics of lattice angles
3:00 – 4:00	Nandor Simanyi	On homotopical rotation vectors of billiards

Combinatorial models for spaces of cubic polynomials

Alexander Blokh, University of Alabama at Birmingham

Abstract: A model for the Mandelbrot set is due to Thurston and is stated in the language of geodesic laminations. The conjecture that the Mandelbrot set is actually homeomorphic to this model is equivalent to the celebrated MLC conjecture stating that the Mandelbrot set is locally connected.

For parameter spaces of higher degree polynomials, even conjectural models are missing, one possible reason being that the higher degree analog of the **MLC-conjecture** is known to be false.

We provide a combinatorial model for an essential part of the parameter space of complex cubic polynomials, namely, for the space of all cubic polynomials with connected Julia sets all of whose cycles are repelling (we call such polynomials *dendritic*).

The description of the model turns out to be very similar to that of Thurston.

An Intrinsic Development of Inversion in Spherical and Hyperbolic Geometries

Robert Foote, Wabash College

Abstract: The theory of circle inversions in the Euclidean plane, E^2 , has a very long history. While the theory can be extended to spherical geometry via stereographic projection and to hyperbolic geometry via the Poincaré models, it loses much of its intrinsic meaning. For example, if p and q are inverses across a circle of radius r centered at c , it is not the case that $cp \cdot cq = r^2$ in S^2 or H^2 , and it is not immediately clear what should replace this. We give an intrinsic development of inversion in S^2 and H^2 , including inversion across horocycles and constant distance curves in H^2 . While much of this is elementary, it is instructive to see how a theory that depends heavily on similarity in E^2 can proceed in S^2 and H^2 . (Incidentally, the generalization of $cp \cdot cq = r^2$ is $\lambda(cp) \cdot \lambda(cq) = \lambda(r)^2$, where $\lambda(r) = A(r)/C(r)$, and $A(r)$ and $C(r)$ are, respectively, the area and circumference of a circle of radius r .) This is joint work with Wabash student Xidian Sun.

Spacing statistics of lattice angles

Florin Boca, University of Illinois at Urbana-Champaign

Abstract: Spacing statistics measure the randomness of uniformly distributed sequences, or more generally increasing sequences of finite sets of real numbers. A familiar example arises from directions

of vectors joining a fixed point in the Euclidean plane, with all (or only visible) points of integer coordinates inside balls of fixed center and increasing radius. It turns out that these directions are not randomly distributed, and even the study of their most popular spacing statistics (gap distribution and pair correlation) turn out to be challenging.

The talk will discuss recent progress in the study of the spacing statistics for this type of geometric configuration, comparing in particular the Euclidean and the hyperbolic situations.

On homotopical rotation vectors of billiards

Nandor Simanyi, University of Alabama at Birmingham

Abstract: In a billiard system in $\mathbb{R}^n/\mathbb{Z}^n \setminus \{obstacles\}$ one lifts the billiard orbit to the universal covering space \mathbb{R}^n of $\mathbb{R}^n/\mathbb{Z}^n$, and takes the average displacement vector in \mathbb{R}^n as the rotation vector of the considered orbit. For systems with one obstacle, the topological study of the arising rotation vectors and sets was carried out by A. Blokh, M. Misiurewicz, and N. Simanyi in 2006. The next step is to consider 2D billiard system in a billiard table Q with highly non-commutative (hyperbolic) fundamental group $\pi_1(Q)$, and to lift the billiard orbits to the Cayley graph of the group $\pi_1(Q)$, and investigate the following:

In what directions ω and at what speed s can the lifted path converge to a point on the infinite horizon of (the Cayley graph of) the group $\pi_1(Q)$?

The ordered pair (ω, s) will be called the “*homotopical rotation number*” of the investigated orbit. Initial results for some 2D billiards were obtained by L. Goswick and myself in 2011. We present a research plan, joint with C. Moxley, on getting generalizations of those results for some higher-dimensional billiards with intriguing fundamental groups $\pi_1(Q)$.