

Challenge of the Week

Problem #3 - October 9 to October 23, 2015

(a) Let A be the sum of the first one million positive integers (from 1 to 1000000), B the sum of their squares, and C the sum of their cubes. Is the fraction $\frac{AB}{C}$ an integer or a non-integer?

(b) The same problem as in (a), but instead of the first one million integers we consider the first 10^n integers and then calculate the respective numbers A , B , and C . Find all positive integers n for which the fraction $\frac{AB}{C}$ is a positive integer.

(c) How many distinct digits are used in the decimal representation of the integer $\frac{AB}{C}$ from item (b)?

Justify your answer for each of the parts (a), (b), and (c) .

Direct any questions to Gregory Galperin, OM 3361.

Rules and Awards

- Any undergraduate currently enrolled at EIU is eligible to participate.
- Each solution is to be the work of one individual and is to be submitted with the solver's name, year in school, email address, local address and home address.
- Each solution is to be written or typed and is due in the main Mathematics Department office (OM3611) by 2:00 p.m., Friday, October 23, 2015.
- Entries will be judged on the basis of clarity of exposition and elegance of solution. That is to say, the *explanation* is more important than the answer.
- An Award of \$25 will be given for the best solution. In the case of a two-way tie, the award will be evenly split. If there are more than two 'best' solutions, a drawing will be held for the reward. In the case no award is made for this challenge, \$25 will be added to the next challenge.
- Challenges, solutions, names of all solvers, and comments will be posted on the Challenge of the Week homepage:
<http://www.eiu.edu/math/challenge.php>.