

University Math Challenge

September 12, 2023 to October 13, 2023

PROBLEM # 1

Some number $n > 10$ boxes numbered, in order, from #1 to # n , are arranged around a circle. Each box contains some number of small balls inside it. Among them, it is known that there is a box with 8 balls, and that there is another box with either 17 or 23 balls, though which of these is unknown. You make the following n transfers from box to box. First you move 1 ball from box #1 to box #2; then move 2 balls from box #2 to box #3; then move 3 balls from box #3 to box #4; and so on. The last two transfers are the following: you move $(n - 1)$ balls from box # $(n - 1)$ to box # n , and, finally, you move n balls from box # n to box #1.

It is known that after making all the n transfers, the number of balls in each box is the same.

Answer the following questions, with justification:

- Which box contained 8 balls in the beginning?
- How many balls were in the other box at the beginning, 17 or 23?
- How many boxes were there? In other words, find n (all possible values).
- How many balls were in each box in the very end (after making all the n transfers)?

*Direct any questions to
Grant Lakeland (OM 3226)*

Rules & Rewards

- Any undergraduate currently enrolled at EIU is eligible to participate.
- Each solution is to be the work of one individual and is to be submitted with the solver's name, year in school, email address, local address, and home address.
- Each solution is to be written or typed and is due in the main Mathematics Department office (OM 3611) by 2:00pm, Friday, October 13, 2023.
- Entries will be judged on the basis of clarity of exposition and elegance of the solution. That is to say, the *explanation* is more important than the answer.
- An award of \$50 will be given for the best solution. In the case of a two-way tie, the award will be evenly split. If there are more than two 'best' solutions, a drawing will be held for the reward. In the case no award is made for this week's challenge, \$50 will be added to the next week's award.
- Names of all solvers will be posted on the Challenge of the Month bulletin board and on the Challenge homepage: <http://www.eiu.edu/math/challenge.php>